(7 pages) **Reg. No. :** ....................................

**Code No. : 7130** **Sub. Code : PPHM 21**

M.Sc. (CBCS) DEGREE EXAMINATION,
APRIL 2018.

Second Semester

Physics

MATHEMATICAL PHYSICS – II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The Cauchy-Riemann equation in polar form is ————

(a) $\frac{\partial u}{\partial r} = \frac{\partial u}{\partial \theta}$ (b) $\frac{\partial u}{\partial r} = -\frac{1}{r}\frac{\partial v}{\partial \theta}$

(c) $\frac{\partial u}{\partial r} = -\frac{\partial v}{\partial \theta}$ (d) $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$

2. If a function at a point is single valued and has a derivative at every point in some neighbourhood of it in a domain H is called ———— function

(a) complex (b) holomorphic

(c) irregular (d) none of these

3. The elements of the smallest set capable of generating all the elements of the group are called ———— of the group

(a) Inverse element (b) Identity element

(c) Generator (d) Reciprocal element

4. The dimensionality theorem can be expressed as ————

(a) $\sum li^2 = n$ (b) $\sum li^2 \le n$

(c) $\sum li^2 \ge n$ (d) $\sum li^2 = 0$

5. $P_{2m+1}(0) =$ ————

(a) 1 (b) 0

(c) $(-1)^m \frac{2m!}{2^{2m}(m!)^2}$ (d) $\frac{2m!}{2^{2m}(m!)^2}$

6. $H_n(0) =$ ————, if $n$ is an odd integer

(a) $(-1)^n \frac{n!}{(n/2)!}$ (b) $\frac{n!}{(n/2)!}$

(c) 0 (d) $(-1)^n$

7. In the three dimensional heat flow equation $\nabla^2 u = \frac{1}{h^2}\frac{\partial u}{\partial t}, h^2$ stands for ———— constant

(a) Planck's (b) diffusivity

(c) Helmholtz (d) Laplacian

8. The equation $\frac{\partial^2 \tau}{\partial t^2} + w^2 \tau = 0$ is the ———— equation of zeroth order

(a) Laguerre's (b) Lagendre

(c) Hermite (d) Bessel

9. Using Kronecker delta, $\delta_\upsilon^\mu A^\mu =$ ————

(a) $A^\mu$ (b) 0

(c) $A^\upsilon$ (d) =1

10. If $A^\mu$ and $B_\mu$ are any two vectors, one contra variant and other covariant, then $A^\mu B_\mu$ is ————

(a) Covariant

(b) Contra variant

(c) Mixed tensor

(d) Invariant



PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) If $f(z)$ is analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$.

Or

(b) State and prove Cauchy's integral formula.

12. (a) Write a short note on cosets.

Or

(b) Write a short note on reducible and irreducible representations and prove that the two dimensional representations of matrices $C_4$ is reducible.

13. (a) Derive the generating function of Legendre polynomial.

Or

(b) Prove that $2xH_n(x) = 2n\, H_{n-1}(x) + H_{n+1}(x)$.

14. (a) The ends A and B of a rod 20 cm long are at temperature 30°C and 80°C respectively. Until steady state prevails. The temperatures at the ends are changed to 40°C and 60°C respectively. Find the temperature distribution in the rod at time t.

Or

(b) Derive the D'Alembert's solution of vibrating string.

15. (a) Elaborate with suitable example the outer product and contraction of tensors.

Or

(b) Derive the expression for strain, stress and Hooke's law in the form of tensors.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Discuss in detail the necessary and sufficient condition for a function to be analytic.

Or

(b) (i) Prove that $u = x^2 - y^2$ and $v = \frac{y}{x^2 + y^2}$ are harmonic functions of $x$ and $y$, but are not harmonic conjugates.

(ii) Prove that the function $f(z) = e^{\sin z}$ is analytic $z = x + iy$.



17. (a) Elaborate in detail the isomorphism and homomorphism.

Or

(b) (i) Prove that two right cosets of a subgroup in a given group are either equal or else have no elements in common.

(ii) Write a short note on conjugate and normal subgroups.

18. (a) Derive the power series solution of Legendre differential equation in descending powers of $x$.

Or

(b) Using the Hermite polynomial of degreen, derive $H_3(x)$ and $H_4(x)$.

19. (a) A thin rectangular plate whose surface is impervious to heat flow has arbitrary distribution of temperature $f(x, y)$ at $t = 0$, Its four edges $x = 0, x = a, y = 0, y = b$ are kept at zero temperature. Determine the subsequent temperature of the plate after time $t$.

Or

(b) Derive the complete solution for the vibrations of a rectangular membrane.

20. (a) Show that $T = \begin{pmatrix} -x_1 x_2 & -x_2^2 \\ x_1^2 & x_1 x_2 \end{pmatrix}$ is a second order tensor in two dimensions and $S = \begin{pmatrix} -x_1 x_2 & -x_2^2 \\ x_1^2 & -x_1 x_2 \end{pmatrix}$ is not a tensor.

Or

(b) Elaborate the applications of tensor to non-relativistia physics using the tensors in the dynamics of a particle.

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