(6 Pages)

Reg. No. : ....

Code No. : 30568 E **Sub. Code : AMMA 41**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2022

Fourth Semester

Mathematics — Core

ABSTRACT ALGEBRA

(For those who joined in July 2020 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. In $(z_7 - \{0\}, \odot\ )$ the inverse of 3 is ————

   (a) 2 (b) 3

   (c) 4 (d) 5

2. If $f : Z \rightarrow Z$ is defined as $f(x) = 2x+1$, which of the following is true?

   (a) f is onto (b) f is $1-1$

   (c) f^{-1} exists (d) f^{-1} does not exist

3. The set of generators of the group $(Z_6, \oplus)$ is ————

   (a) {1, 5} (b) {1, 2, 4}

   (c) {1, 2, 5} (d) {2, 3, 5}

4. In the group S_4, the inverse of (1 2 3 4) is ————

   (a) (1 2 3 4) (b) (1 2 4 3)

   (c) (1 4 3 2) (d) (1 3 2 4)

5. The Kernel of the homomorpshism $f : (Z, +) \rightarrow (R^+, \bullet)$ defined by $f(x) = 2^x$ is ————

   (a) {1} (b) Z

   (c) {–1, 1} (d) {0}

6. S_2 is ————

   (a) not a group

   (b) infinite group

   (c) not an abelian group

   (d) abelian group

7. An example of an infinite commutative ring without identity is ————

   (a) $(Z, +, \cdot)$ (b) $(Z_n, \oplus, \odot\ )$

   (c) $(2Z, +, \cdot)$ (d) $(Q, +, \cdot)$

8. If I is an ideal of a ring R with identity 1 and if $1 \in I$, then $I =$ ———

(a) {1} (b) {0}

(c) {0, 1} (d) R

9. Field of quotients of Z is ———

(a) Q (b) N

(c) Z (d) Q^C

10. If Z_n is an integral domain n is ———

(a) an integer (b) 0

(c) composite number (d) prime number

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let G be a group and $H = \{a / a \in G,\ ax = xa \forall x \in G\}$. Prove that H is a subgroup of G.

Or

(b) Let G be a group and 'a' be an element of order 'n'. Show that $a^m = e$ iff n divides m.

12. (a) In $G = R - \{-1\}$, if we define $a * b = a + b + ab$. Prove that $(G, *)$ is a group.

Or

(b) Prove that a non-empty subset H of a group G is a subgroup if and only if $ab^{-1} \in H \forall a, b \in H$.

13. (a) Prove that every subgroup of an abelian group is normal.

Or

(b) Let $f : G \to G'$ be a homomorphism. Prove that if f is 1 – 1 then $\ker f = \{e\}$.

14. (a) If R is a ring and $a, b \in R$, prove the following

(i) $a.0 = 0.a = 0$

(ii) $a(-b) = (-a)b = -ab$

(iii) $(-a)(-b) = ab$.

Or

(b) Show that $\left\{\begin{pmatrix} a & b \\ -b & a \end{pmatrix} \Big/ a, b \in R\right\}$ is a ring under matrix addition and multiplication.

15. (a) Prove that the only isomorphism $f: Q \to Q$ is the identity map.

Or

(b) Prove that $R[x]$ is an integral domain iff R is an integral domain.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Show that a function $f: A \to B$ is a bijection if and only if there is a $g: B \to A$ with $g \circ f = I_A$ and $f \circ g = I_B$.

Or

(b) Show that $f: R - \{3\} \to R - \{1\}$ given by $f(x) = \frac{x-2}{x-3}$ is a bijection. Also find its inverse.

17. (a) If A, B are two subgroups of a group G prove that AB is a subgroup $\Leftrightarrow AB = BA$.

Or

(b) Define a cyclic group. Prove that every finite cyclic group f order n is isomorphic to the additive group of residue classes modulo n.

18. (a) Let G be a group and $a, b \in G$. Prove the following

(i) orders of a, a^{-1} are the same

(ii) a and $b^{-1}ab$ have the same order

(iii) ab and ba have the same order.

Or

(b) If $f: G \to G'$ is a homomorphism with Kernel K, prove that $\frac{G}{K} \cong f(G)$.

19. (a) Prove that $F\{a + b\sqrt{2} : a, b \in Q\}$ is a field under usual addition and multiplication of numbers.

Or

(b) If R is a field, show that the only ideals are $\{0\}$ and R. Is the converse true? Why?

20. (a) Prove that every isomorphic image of a field is a field.

Or

(b) State and prove division algorithm.
