(7 pages)

Reg. No. :

Code No. : 30575 E Sub. Code : SMMA 52

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2020.

Fifth Semester

Mathematics-Core

REAL ANALYSIS – II

(For those who joined in July 2017 onwards)

Time : Three hours

Maximum : 75 marks

PART A —  $(10 \times 1 = 10 \text{ marks})$ 

Answer ALL questions.

Choose the correct answer :

- 1. If *d* is a metric on *M*, which one of the following is not a metric on *M*?
  - (a)  $d^2$  (b)  $\sqrt{d}$
  - (c)  $\frac{d}{1+d}$  (d) 2d

- 2. Which one of the following is not correct?
  - (a)  $\operatorname{int}(A \cup B) \supseteq \operatorname{int} A \cup \operatorname{int} B$
  - (b)  $\operatorname{int}(A \cup B) \subseteq \operatorname{int} A \cup \operatorname{int} B$
  - (c)  $\operatorname{int}(A \cap B) \supseteq \operatorname{int} A \cap \operatorname{int} B$
  - (d)  $\operatorname{int}(A \cap B) \subseteq \operatorname{int} A \cap \operatorname{int} B$
- 3. In the usual metric space (R,d), the limit point of

$$\begin{cases} 1, \frac{1}{2}, \frac{1}{3}, \dots \end{cases} is$$
(a)  $\infty$  (b) 1
(c)  $\frac{1}{2}$  (d) 0

- 4. Which one of the following is not of second category?
  - (a) [a,b] (b) (a,b)
  - (c) [a,b) (d) Q
- 5. f is continuous at c iff

(a) 
$$f(x) = f(c)$$
 (b)  $\lim_{x \to c} f(x) = f(c)$ 

(c) f(x) = c (d)  $\lim_{x \to c} f(x) = c$ 

Page 2 Code No. : 30575 E

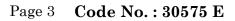
6.	If the function $f: R \to R$ is defined by $f(x) = [x]$
	then $w(f, 10) =$

- 7. In R with discrete metric, which is connected?
  - (a) R (b) [a, b]
  - (c)  $(0,\infty)$  (d) {1}
- 8. With usual metric R is
  - (a) not complete
  - (b) compact
  - (c) connected
  - (d) connected and compact

9. 
$$\int_{-1}^{1} |x| dx =$$
(a)  $\frac{1}{2}$  (b) 0
(c) 2 (d) 1
10. 
$$\int_{-1}^{t} \sin x \, dx =$$

10. 
$$\int_{0} \sin x \, dx$$

(a)	$\cos t - 1$	(b)	$1 - \cos t$
(c)	$1 + \cos t$	(d)	$-1-\cos t$



PART B —  $(5 \times 5 = 25 \text{ marks})$ 

Answer ALL questions, choosing either (a) or (b).

11. (a) Let (M, d) be a metric space. Define  $d_1(x, y) = \min \{1, d(x, y)\}$ . Prove that  $d_1$  is a metric on M.

## Or

- (b) Show that the intersection of finite collection of open sets is open.
- 12. (a) In R with usual metric prove that D[0,1) = [0,1].

## Or

- (b) Let (M,d) be a metric space and  $A \subseteq M$ . If  $x \in \overline{A}$  prove that there exists a sequence  $(x_n)$  in A such that  $(x_n) \to x$ .
- 13. (a) If  $f: m_1 \to m_2$  and  $g: m_2 \to m_3$  are continuous functions prove that  $g \circ f: m_1 \to m_3$  is continuous.

## Or

(b) Prove that  $f: m_1 \to m_2$  is continuous iff  $f^{-1}(F)$  is closed in  $m_1$  whenever F is closed in  $m_2$ .

Page 4 Code No. : 30575 E [P.T.O.] 14. (a) State and prove intermediate value theorem.

Or

- (b) Prove that any closed subspace of a compact metric space is compact.
- 15. (a) Show that  $x^2$  is integrable on any interval [0, K].

Or

(b) State and prove fundamental theorem of calculus.

PART C —  $(5 \times 8 = 40 \text{ marks})$ 

Answer ALL questions, choosing either (a) or (b).

16. (a) d and P are metrices on m. If there exists K > 1 such that  $\frac{1}{K}P(x, y) \le d(x, y) \le KP(x, y)$  for all  $x, y \in m$  prove that d and P are equivalent.

- Or
- (b) Let (m,d) be a metric space. If  $A \subseteq m$  prove that int A is the largest open set contained in A.

Code No. : 30575 E Page 5

17. (a) Prove that a necessary and sufficient condition for a subset A of a complete metric space m to be complete is that A is closed.

# $\mathbf{Or}$

- (b) Prove that for any subset A of a metric space,  $d(A) = d(\overline{A})$
- 18. (a) Let  $(m_1, d_1)$  and  $(m_2, d_2)$  be metric spaces. Let  $a \in m_1$ . prove that a function  $f: m_1 \to m_2$ is continuous at a iff  $(x_n) \to a \Rightarrow (f(x_n)) \to f(a)$ .

# Or

- (b) If D is the set of points of discontinuities of a function  $f: R \to R$  show that D is of type  $F_{\sigma}$ .
- 19. (a) Let A be a connected subset of a metric space m. If B is a subset of m such that  $A \subseteq B \subseteq \overline{A}$  prove that B is also connected.

 $\mathbf{Or}$ 

(b) Prove that any compact subset of a metric space is closed.

Page 6 Code No. : 30575 E

20. (a) State and prove Taylor's theorem.

Or

- (b) (i) State and prove Lagrange's mean value theorem.
  - (ii) State and prove Cauchy's mean value theorem.

Page 7 Code No. : 30575 E