(7 pages) **Reg. No. :** ....

Code No. : 30575 E **Sub. Code : SMMA 52**

B.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2020.

Fifth Semester

Mathematics – Core

REAL ANALYSIS – II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If d is a metric on M, which one of the following is not a metric on M?

   (a) d^2 (b) $\sqrt{d}$

   (c) $\frac{d}{1+d}$ (d) $2d$

2. Which one of the following is not correct?

(a) $\text{int}(A \cup B) \supseteq \text{int}\, A \cup \text{int}\, B$

(b) $\text{int}(A \cup B) \subseteq \text{int}\, A \cup \text{int}\, B$

(c) $\text{int}(A \cap B) \supseteq \text{int}\, A \cap \text{int}\, B$

(d) $\text{int}(A \cap B) \subseteq \text{int}\, A \cap \text{int}\, B$

3. In the usual metric space (R, d), the limit point of $\left\{1, \frac{1}{2}, \frac{1}{3}, \ldots\right\}$ is

(a) ∞ (b) 1

(c) $\frac{1}{2}$ (d) 0

4. Which one of the following is not of second category?

(a) $[a, b]$ (b) (a, b)

(c) $[a, b)$ (d) Q

5. f is continuous at c iff

(a) $f(x) = f(c)$ (b) $\lim_{x \to c} f(x) = f(c)$

(c) $f(x) = c$ (d) $\lim_{x \to c} f(x) = c$

6. If the function $f: R \to R$ is defined by $f(x) = [x]$ then $w(f, 10) =$

(a) 2 (b) 0

(c) 1 (d) 4

7. In R with discrete metric, which is connected?

(a) R (b) $[a, b]$

(c) $(0, \infty)$ (d) $\{1\}$

8. With usual metric R is

(a) not complete

(b) compact

(c) connected

(d) connected and compact

9. $\int_{-1}^{1} |x| dx =$

(a) $\frac{1}{2}$ (b) 0

(c) 2 (d) 1

10. $\int_{0}^{t} \sin x \, dx =$

(a) $\cos t - 1$ (b) $1 - \cos t$

(c) $1 + \cos t$ (d) $-1 - \cos t$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let (M, d) be a metric space. Define $d_1(x, y) = \min\{1, d(x, y)\}$. Prove that d_1 is a metric on M.

Or

(b) Show that the intersection of finite collection of open sets is open.

12. (a) In R with usual metric prove that $D[0, 1) = [0, 1]$.

Or

(b) Let (M, d) be a metric space and $A \subseteq M$. If $x \in \overline{A}$ prove that there exists a sequence (x_n) in A such that $(x_n) \to x$.

13. (a) If $f : m_1 \to m_2$ and $g : m_2 \to m_3$ are continuous functions prove that $g \circ f : m_1 \to m_3$ is continuous.

Or

(b) Prove that $f : m_1 \to m_2$ is continuous iff $f^{-1}(F)$ is closed in m_1 whenever F is closed in m_2.

14. (a) State and prove intermediate value theorem.

Or

(b) Prove that any closed subspace of a compact metric space is compact.

15. (a) Show that x^2 is integrable on any interval $[0, K]$.

Or

(b) State and prove fundamental theorem of calculus.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) d and P are metrices on m. If there exists $K > 1$ such that $\frac{1}{K} P(x, y) \leq d(x, y) \leq KP(x, y)$ for all $x, y \in m$ prove that d and P are equivalent.

Or

(b) Let (m, d) be a metric space. If $A \subseteq m$ prove that $\operatorname{int} A$ is the largest open set contained in A.

17. (a) Prove that a necessary and sufficient condition for a subset A of a complete metric space m to be complete is that A is closed.

Or

(b) Prove that for any subset A of a metric space, $d(A) = d(\overline{A})$

18. (a) Let (m_1, d_1) and (m_2, d_2) be metric spaces. Let $a \in m_1$. prove that a function $f : m_1 \to m_2$ is continuous at a iff $(x_n) \to a \Rightarrow (f(x_n)) \to f(a)$.

Or

(b) If D is the set of points of discontinuities of a function $f : R \to R$ show that D is of type F_σ.

19. (a) Let A be a connected subset of a metric space m. If B is a subset of m such that $A \subseteq B \subseteq \overline{A}$ prove that B is also connected.

Or

(b) Prove that any compact subset of a metric space is closed.

20. (a) State and prove Taylor's theorem.

Or

(b) (i) State and prove Lagrange's mean value theorem.

(ii) State and prove Cauchy's mean value theorem.

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