(6 Pages) **Reg. No. :** ....

Code No. : 30345 E **Sub. Code : SMMA 62**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2022

Sixth Semester

Mathematics — Core

NUMBER THEORY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

1. The value of $1^2+3^2+5^2+\cdots+(2n-1)^2=$

(a) $\binom{2n+2}{3}$ (b) $\binom{2n}{n}$

(c) $\binom{2n}{3}$ (d) $\binom{2n+1}{3}$

2. If n is an odd integer, and $r=\frac{1}{2}(n-1)$, then

(a) $\binom{n}{r}=\binom{n}{r-1}$ (b) $\binom{n}{r}=\binom{n+1}{r+1}$

(c) $\binom{n}{r}=\binom{n}{r+1}$ (d) $\binom{n+1}{r}=\binom{n+1}{r+1}$

3. $lcm(3054, 12378)=$ ————

(a) 6300402 (b) 3054

(c) 12378 (d) 6

4. Given indegers a,b,c,d, which one of the following is false?

(a) If $a\mid bc$ then $a\mid c$

(b) If $a\mid b$ and $a\mid c$ then $a^2\mid bc$

(c) $a\mid b$ if and only if $ac\mid bc$, where $c=0$

(d) If $a\mid b$ and $c\mid d$ then $ac\mid bd$

5. Prime factorization of 17460 is ————

(a) 8.9.5.49 (b) $2^3.3^2.5.7^2$

(c) $2^3.3.5.7^3$ (d) $8.9.5.7^2$

6. For $n \geq 2$, $\sqrt[n]{n}$ is ————

(a) irrational (b) rational

(c) composite (d) integer

7. The remainder of $2^{20} - 1$ is divisible by 41 ————

(a) 1 (b) 2

(c) 3 (d) 0

8. Number of solutions of $18x \equiv 30 (\mathrm{mod}\, 42)$ is ————.

(a) 2 (b) 6

(c) 3 (d) 5

9. Any absolute pseudoprime is ————

(a) square free (b) pseudo prime

(c) prime (d) absolute

10. The unit digit of 3^{100} is ————

(a) 0 (b) 1

(c) 2 (d) 3

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $1.2 + 2.3 + 3.4 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$, for all $n \geq 1$.

Or

(b) Derive the Binomial identity $\binom{2}{2} + \binom{4}{2} + \binom{6}{2} + \cdots + \binom{2n}{2} = \frac{n(n+1)(4n-1)}{6}$ $n \geq 2$.

12. (a) Show that the expression $a(a^2+2)/3$ is an integer for all $a \geq 1$.

Or

(b) For any integers a, b prove that if $a \mid b$ and $b \neq 0$ then $|a| \leq |b|$.

13. (a) State and prove Euclid's theorem.

Or

(b) Employing the Sieve of eratosthenes, obtain all the primes between 100 and 200.

14. (a) If $ca \equiv cb (\bmod n)$ then prove that $a \equiv b (\bmod n/d)$, where $d = \gcd(c, n)$.

Or

(b) Find the remainder when $1! + 2! + \cdots + 100!$ is divided by 12.

15. (a) State and prove Wilson's theorem.

Or

(b) If p and q are distinct primes with $a^p \equiv a (\bmod p)$ and $a^q \equiv a (\bmod q)$ then prove that $a^{pq} \equiv a (\bmod pq)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove first principle of induction.

Or

(b) Prove that the sum of the reciprocals of the first 'n' triangular numbers is less than 2.

17. (a) State and prove division algorithm.

Or

(b) Find the solution of linear diophantine equation $24x + 138y = 18$.

18. (a) If all the $n > 2$ terms of the arithmetic progression $p, p+d, \ldots p+(n-1)d$ are prime numbers then prove that the common difference d is divisible by every prime $q < n$.

Or

(b) State and prove Fundamental theorem of Arithmetic.

19. (a) State and prove Chineese remainder theorem.

Or

(b) Find the solutions of the system of congruences $3x + 4y \equiv 5 (\bmod 13)$, $2x + 5y \equiv 7 (\bmod 13)$.

20. (a) State and prove Fermat's theorem.

Or

(b) Prove that the quadratic congruence $x^2 + 1 \equiv 0 (\bmod p)$, where p is an odd prime has a solution iff $p \equiv 1 (\bmod 4)$.

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