(8 pages) **Reg. No. :** .....

Code No. : 10725 E **Sub. Code : EMMA 21**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2024.

Second Semester

Mathematics – Core

ANALYTICAL GEOMETRY (TWO AND THREE DIMENSIONS)

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The polar of $(ae,0)$ with respect to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is _____

(a) $x=a$ (b) $x=ab$

(c) $x=\frac{a}{e}$ (d) $x=ae$

2. The limes $y=mx$ and $y=m,x$ are conjugate diameters of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, then $mm_1=$ _____

(a) $-\frac{b^2}{a^2}$ (b) $\frac{b^2}{a^2}$

(c) $-\frac{a^2}{b^2}$ (d) $\frac{a^2}{b^2}$

3. The equation of the line through the pole is _____

(a) $r=\theta$ (b) r a constant

(c) $\theta=$ a constant (d) $r=0$

4. The condition under which the line $\frac{l}{r}=A\cos\theta+B\sin\theta$ is a tangent to the conic $\frac{l}{r}=1+e\cos\theta$ is _____

(a) $A^2+B^2+e^2$ (b) $(A-e)^2+B^2=1$

(c) $A^2+(B+e)^2=1$ (d) $A+B=e$

5. The distance from the origin to the plane $6x-3y+2z-14=0$ is ———— units.

(a) 14 (b) 2

(c) 7 (d) 6

6. The equation of the plane perpendicular to the plane $2x-y+5z-2=0$ is ————

(a) $2x-y+5z+2=0$ (b) $3x+y+z-2=0$

(c) $3x+y-z+2=0$ (d) $-2x+y-5z+2=0$

7. A point on the line $\frac{x-2}{3}=\frac{y+4}{1}=\frac{z}{-1}$ is ————.

(a) $(2,-4,0)$ (b) $(3,1,-1)$

(c) $(2,4,0)$ (d) $(-2,4,0)$

8. The plane ———— is perpendicular to the line $\frac{x+2}{-1}=\frac{y-3}{1}=\frac{z+4}{3}$.

(a) $-x+y+3z=4$ (b) $2x-3y+4z=1$

(c) $-2x+3y-4z=5$ (d) $x-y-3z=1$

9. The radius of the sphere $x^2+y^2+z^2-6x-2y-4z-11=0$ is ————

(a) 11 (b) 7

(c) 5 (d) 6

10. If the plane $lx+my+nz=p$ is a tangent plane to the sphere $x^2+y^2+z^2=r^2$ then $l^2+m^2+n^2=$ ————

(a) $\frac{p^2}{r^2}$ (b) $\frac{r^2}{p^2}$

(c) p^2 (d) r^2

PART B — (5 × 5 = 25 marks)

Answer ALL questions by choosing (a) or (b).

11. (a) Find the locus of the poles of normal chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Or

(b) Explain about the equi- conjugate diameters of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

12. (a) Derive the polar equation of a straight line.

Or

(b) Find the locus of the foot of the perpendiculars drawn from the pole to the tangents to the circle $r = 2a\cos\theta$.

13. (a) Find the equation of the plane passing through the points $(3,1,2)$ and $(3,4,4)$ and perpendicular to the plane $5x + y + 4z = 0$.

Or

(b) Find the ratio in which the plane $ax + by + cz + d = 0$ divides the line joining the points (x, y, z) and (x_2, y_2, z_2).

14. (a) Find the symmetrical form of the equations of the line of intersection of the planes $x + 5y - z - 7 = 0$ and $2x - 5y + 3z + 1 = 0$.

Or

(b) Obtain the value of λ if the lines $\frac{x+2}{4} = \frac{y+1}{2} = \frac{z+\lambda}{3}$ intersects the line $\frac{x-6}{2} = \frac{y-5}{3} = \frac{2z-7}{2}$.

15. (a) Find the equation of the sphere which passes through the points $(2,3,1), (5,-1,2), (4,3,-1)$ and $(2,5,3)$.

Or

(b) Find the equation of the sphere which touches the sphere $x^2 + y^2 + z^2 - 6x + 2z + 1 = 0$ at the point $(2,-2,1)$ and passes through the origin.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions by choosing (a) or (b).

16. (a) Show that the conjugate lines through a focus of all ellipse are at right angles.

Or

(b) If e and e_1 are the eccentricities of a hyperbola and its conjugate, then prove: $e^2 + e_1^2 = e^2 . e_1^2$.

17. (a) Find the equation of the chord joining the points whose vertical angles are θ_1 and θ_2 on the circle $r = 2a\cos\theta$.

Or

(b) Show that the locus of the intersection of perpendicular tangents to a conic is a circle.

18. (a) The foot of the perpendicular drawn from the origin to the plane is $(12,-4,-3)$. Find the equation of the plane.

Or

(b) Find the equations of the planes passing through the line of intersection of the planes $5x-3y+4=0$ and $x+y-2z+10=0$ and which are 1 unit distance from the origin.

19. (a) Obtain the image of the point $(1,-2,3)$ in the plane $2x-3y+2z+3=0$.

Or

(b) Prove that the lines $\frac{x+1}{-3}=\frac{y+10}{8}=\frac{z-1}{2}$ and $\frac{x+3}{-4}=\frac{y+1}{7}=\frac{z-4}{1}$ are coplanar. Find their point of intersection. Also find the plane through them.

20. (a) A plane passes through a fixed point (a,b,c) and cuts the axes in A,B,C respectively. Show that the locus of the centre of the sphere passing through the points O,A,B,C is $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$.

Or

(b) Derive the condition that the line $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n}$ touches the sphere $x^2+y^2+z^2+2ux+2uy+2wz+d=0$ if $l^2+m^2+n^2=1$.
