

(7 pages)

Reg. No. :

Code No. : 9370

Sub. Code : HMAE 41

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2015.

Fourth Semester

Mathematics

Elective – GRAPH THEORY

(For those who joined in July 2012 onwards)

Time : Three hours

Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. Which of the following is true?

- (a) An empty graph contains only one point
- (b) Every walk is a path
- (c) Subgraph of a connected graph is connected
- (d) There is a 4-regular graph on 7 vertices

2. Which of the following is true?

- (a) Every cycle is a path
- (b) A bipartite graph always has an even cycle
- (c) If a graph is acyclic, then it is a tree
- (d) Number of spanning trees of a tree on v vertices is 1

3. Which of the following is true?

- (a) A connected graph always has only one block
- (b) A tree has only one block
- (c) In a connected graph any two vertices lie on a common cycle
- (d) If G is a block having 4 vertices, then any two edges lie on a common cycle

4. Which of the following is true?

- (a) A tree will never be an Euler graph
- (b) K_6 is both Eulerian and Hamiltonian
- (c) Every Hamiltonian graph is Eulerian
- (d) Every tree is Hamiltonian

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5. Which of the following is true?
- (a) Every graph has a perfect matching
 - (b) $K_{4,3}$ has a perfect matching
 - (c) If G is bipartite, then $\chi'(G) = \Delta - 1$
 - (d) K_{12} has a perfect matching
6. Which of the following is true?
- (a) $K_{n,n+2}$ has a perfect matching
 - (b) Every 3-regular graph has a perfect matching
 - (c) A cycle on 9 vertices has a perfect matching
 - (d) Every graph on $2n$ vertices has a perfect matching
7. Which of the following is true?
- (a) K_n has no independent set
 - (b) Covering number may be equal to the independence number
 - (c) Every independent set is contained in a maximum independent set
 - (d) The Ramsey number $r(2, n) = 2$

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8. Which of the following is true?
- (a) $K_{3,3}$ has an independent set on 2 vertices
 - (b) independence number is always less than or equal to the covering number
 - (c) For any graph on v vertices the independence number is less than v
 - (d) The Ramsey number $r(3, 3) = 5$
9. Which of the following is true?
- (a) If G is k -critical, then $\delta \leq k - 1$
 - (b) If G is k -critical, then $\delta \geq k$
 - (c) Chromatic number of a cycle on n vertices is n
 - (d) None of the above
10. Which of the following is true?
- (a) Every 4-chromatic graph has $4n$ vertices
 - (b) K_5 has a subdivision of K_4 as a subgraph.
 - (c) K_5 can be obtained as a subdivision of K_4
 - (d) None of the above

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[P.T.O.]



PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that in a group of two or more people, there are always two people having equal number of friends.

Or

- (b) With usual notations show that any graph contains at least $\varepsilon - v + \omega$ distinct cycles.

12. (a) Draw a graph for which the inequality $K \leq K' \leq \delta$ is strict.

Or

- (b) If C is a non-hamiltonian graph with $v \geq 3$, then prove that G is degree majorised by some $C_{m,v}$.

13. (a) In a bipartite graph, show that the number of edges in a maximum matching is the number of vertices in a minimum covering.

Or

- (b) Prove that every 3-regular graph without cut edges has a perfect matching.

14. (a) Prove that a subset S of V is an independent set of G if and only if $V - S$ is a covering of G . Hence or otherwise prove that $\alpha + \beta = v$.

Or

- (b) Show that $r(k, l) \leq \binom{k+l-2}{k-1}$.

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15. (a) Prove that for any graph G , $\chi(G) \leq \Delta + 1$

Or

- (b) If G is simple then prove that $\pi_k(G) = \pi_k(G - e) - \pi_k(G \setminus e)$ for any edge e of G .

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Show that a graph is bipartite if and only if it contains no odd cycle.

Or

- (b) Prove that $\tau(K_n) = n^{n-2}$.

17. (a) Prove that a graph G with $v \geq 3$ is 2-connected if and only if any two vertices of G are connected by at least two internally disjoint paths.

Or

- (b) Prove that a nonempty connected graph is Eulerian if and only if it has no vertices of odd degree.

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18. (a) Show that (i) every k -cube has a perfect matching for $k \geq 2$ (ii) show that a tree has atmost one perfect matching.

Or

- (b) For $k > 0$, show that every k -regular bipartite graph is 1-factorable and every $2k$ -regular graph is 2-factorable.
19. (a) For any two integers $k \geq 2$ and $l \geq 2$, prove that $r(k, l) \leq r(k, l-1) + r(k-1, l)$.

Or

- (b) Write an essay on Ramsey theory.
20. (a) Let G be a k -critical graph with a two vertex cut $\{u, v\}$. Then prove that $d(u) + d(v) \geq 3k - 5$.

Or

- (b) If G is 4-chromatic, then prove that G contains a subdivision of K_4 .
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