(6 pages) **Reg. No. :** ....

Code No. : 6378 **Sub. Code : ZMAM 32**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Third Semester

Mathematics

GRAPH THEORY

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

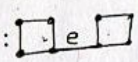
Answer ALL questions.

Choose the correct answer :

1. The number of edges in $K_{4,5}$ is

(a) 9 (b) 20

(c) 1 (d) 45

2. Consider the graph G : 

In $G - e$, the number of vertices is

(a) 2 (b) 9

(c) 6 (d) 8

3. If G is a tree with 16 edges then the number of vertices of G is

(a) 17

(b) 15

(c) 0

(d) any number between 1 and 16

4. Which one of the following is nit true?

It e is link of G then

(a) $\gamma(G \cdot e) = \gamma(G) - 1$

(b) $\varepsilon(G \cdot e) = \varepsilon(G) - 1$

(c) $\omega(G \cdot e) = \omega(G) - 1$

(d) $G \cdot e$ is a tree is G is a tree

5. In the Konigsberg bridge problem, the number of bridge is

(a) 5 (b) 7

(c) 9 (d) 11

6. In $C_{2,5}$, the number of edges is

(a) 7 (b) 10

(c) 3 (d) 5

7. For a graph G with 12 vertices, it $\alpha = 3$ then β is

(a) 15 (b) 36

(c) 9 (d) 3

8. The value of $r(3,3)$ is

(a) 6 (b) 9

(c) 0 (d) 3

9. If G is 5-critical then

(a) $\delta = 5$ (b) $\delta \geq 4$

(c) $\delta \geq 5$ (d) $\delta \leq 4$

10. There are only ———— types of graph G for which $\chi = \Delta + 1$?

(a) 2 (b) 3

(c) 4 (d) 5

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Explain the following with suitable examples

(i) Simple graph

(ii) Spanning subgraph

(iii) Walk

(iv) Path

(v) Cycle.

Or

(b) Prove that $\sum_{v \in V} d(v) = 2\varepsilon$ and hence show the number of vertices of odd degree is even in any graph.

12. (a) If G is a tree, prove that $\varepsilon = \gamma - 1$.

Or

(b) Prove that a vertex v of a tree G is a cut vertex of G if and only if $d(v) > 1$.

13. (a) Prove that $C(G)$ is well defined.

Or

(b) Define a maximum matching and a minimum covering. Let M be a matching k and be a covering such that $|M| = |K|$, then prove that M is a maximum making and K is a minimum covering.

14. (a) If $\delta > 0$, prove that $\alpha' + \beta' = \gamma$.

Or

(b) Prove that $r(k,l) \le \binom{k+l-2}{k-1}$.

15. (a) If G is k-critical, prove that $\delta \ge k - f$.

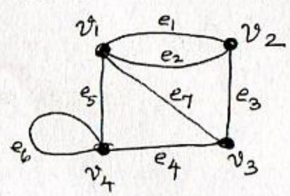
Or

(b) For any graph G, prove that $\prod_k(G)$ is a polynomial is k of degree r, with integer coefficients, leading term k^r and constant term zero and the coefficients of $\prod_k(G)$ alternative in sign.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Define the incidence and adjacency matrices of a graph. Find the two matrices for the following graph.



Or

(b) Obtain a necessary and sufficient condition for graph to be bipartite.

17. (a) Define a cut edge with an example. Prove that an edge e of G is a cut edge of G if and only if e is contained in no cycle of G.

Or

(b) State and prove Whitney's theorem for 2-connected graphs.

18. (a) Let G be a simple graph with degree sequence $(d_1, d_2, \dots dr)$ when $d_1 \le d_2 \le \dots \le d_r$ and $r \ge 3$. Suppose that there is no value of m less than $r/2$ for which $d_m \le m$ and $d_{r-m} < r - m$. Prove that G is Hamiltonian.

Or

(b) Prove that a matching M in G is a maximum matching if and only if G contains no M-augmenting path.

19. (a) If G is simple, prove either $\chi' = \Delta\theta\chi' = \Delta + 1$.

Or

(b) Prove that $r(k,k) \ge 2^{k/2}$.

20. (a) For any positiver integer k, prove that there exists a k-chromatic graph containing no triangle.

Or

(b) State and prove Brook's theorem.
