(6 pages) **Reg. No. :** ....

Code No. : SS 30573 E **Sub. Code : SMMA 41**

B.Sc. (CBCS) DEGREE (Special Supplementary)
EXAMINATION, APRIL 2020.

Fourth Semester

Mathematics – Core

ABSTRACT ALGEBRA – I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. For any two elements $a, b \in G$ the statement $(ab)^2 = a^2 b^2$ is true if ————

   (a) G is abelian (b) G is non abelian

   (c) G is any group (d) G is finite

2. The order of an element 2 in $(z, +)$ is ————

   (a) 2 (b) infinite

   (c) 1 (d) –2

3. Let G be a group of prime order. Then

(a) G has no subgroup

(b) G has no proper subgroups

(c) G has more than 2 subgroups

(d) G is non abelian

4. Subgroup of a cyclic group is ————

(a) not cyclic (b) a subgroup

(c) cyclic (d) none

5. The order of the group $\frac{z_6}{\langle 3 \rangle}$ is ————

(a) 1 (b) 2

(c) 3 (d) Infinite

6. Aut $Z_8 \cong$ ————

(a) V_4 (b) V_2

(c) Z_4 (d) V_8

7. Every subgroup of $(z_n, \oplus)$ is ————

(a) a subgroup (b) normal group

(c) prime (d) none

8. An example of an infinite commutative ring without identify

(a) $(Z,+,\cdot)$ (b) $(Z_n,\oplus,\odot)$

(c) $(2Z,+,\cdot)$ (d) $(Q,+,\cdot)$

9. Let R be a ring and $a \in R$. Then $Ra = \{xa / x \in R\}$ is a ————

(a) right ideal (b) left ideal

(c) ideal (d) None

10. A homomorphism is $1-1 \Leftrightarrow$ its Kernal is ————

(a) $\{0\}$ (b) $\{1\}$

(c) $\frac{\langle 0 \rangle}{R}$ (d) none

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let G be a group. Let $H_a = \{x / x \in G \text{ and } ax = xa\}$. Then prove that H_a is a subgroup of G.

Or

(b) Prove that $(Z_n,\oplus)$ is a group.

12. (a) Prove that a subgroup of a cyclic group is cyclic.

Or

(b) Let G be a group and 'a' be an element of order n in G. Show that $a^m = e$ iff n divides m.

13. (a) Show that if a group G has exactly one subgroup H of given order, then H is a normal subgroup of G.

Or

(b) State and prove Fermat's theorem.

14. (a) Prove that a finite commutative ring R without zero divisors is a field.

Or

(b) Prove that the characteristic of an integral domain is either 0 or a prime number.

15. (a) Define homomorphism of rings. Give an example.

Or

(b) Define Kernal of a homomorphism. Let $f : R \to R'$ be a homomorphism. Let K be the Kernal of f. Then prove that K is an ideal of R.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Let A and B be two subgroups of a group G. Prove that AB is a subgroup of G if and only if $AB = BA$.

Or

(b) Prove that the union of two subgroups of a group G is a subgroup if and only if one is contained in the other.

17. (a) Let G be a group and $a, b \in G$. Then prove the following :

(i) order of a = order of a^{-1}

(ii) order of a = order of $b^{-1}ab$

(iii) order of ab = order of ba.

Or

(b) Let G be a group and H be a subgroup of G. Then prove the following :

(i) $a \in H \Rightarrow aH = H$

(ii) $aH = bH \Rightarrow a^{-1}b \in H$

(iii) $a \in bH \Rightarrow a^{-1} \in Hb^{-1}$

(iv) $a \in bH \Rightarrow aH = bH$.

18. (a) State and prove the fundamental theorem of homomorphism for groups.

Or

(b) State and prove Cayley's theorem.

19. (a) Prove : Z_n is an integral domain $\Leftrightarrow n$ is a prime.

Or

(b) Let R be a commutative ring with identity prove an ideal P of R is prime $\Leftrightarrow \frac{R}{P}$ is an integral domain.

20. (a) Prove that every integral domain can be embedded in a field.

Or

(b) State and prove division algorithm.