12. (a) State and prove De Morgan's laws for sets.

Or

(b) Using characteristic functions prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

13. (a) For any commutative monoid, prove that the set of idempotent elements forms a sub-monoid.

Or

(b) Show that the Kernel of a homomorphism is a sub group.

14. (a) Show that the sum of in degrees of all the nodes of a simple diagraph is equal to the sum of out degrees of all its nodes and that sum is equal to the number of edges of the graph.

Or

(b) Set T, be a rooted tree on a set A. Then prove that

(i) T is irreflexive and

(ii) T is asymmetric.

15. (a) State and prove absorption laws of Lattices.

Or

(b) Show that in a lattice

$a \leq c \Leftrightarrow a \vee (b \wedge c) (a \vee b) \wedge c$.

PART C — (5 × 8 = 40 marks)

Answer ALL the questions, choosing either (a) or (b).

16. (a) Show the following equivalences :

(i) $A \rightarrow (P \wedge C) \Leftrightarrow (A \wedge 7P) \rightarrow C$

(ii) $(P \rightarrow C) \wedge (Q \rightarrow C) \Leftrightarrow (P \vee Q) \rightarrow C$.

Or

(b) Obtain the disjunctive normal forms of

(i) $P \wedge (P \rightarrow Q)$

(ii) $7(P \vee Q)\ (P \cap Q)$.

17. (a) Show that the set of all the invertible elements of a monoid form a group under the same operation as that of the monoid.

Or

(b) State and prove the Lagrange's theorem.

18. (a) Describe Washall's algorithm.

Or

(b) Show that the function $f: R \to R$ defined by $f(x) = 3x - 1$, $x \in R$ is a bijection.

19. (a) Show that in a Boolean algebra $(a * b)^1 = a^1 \oplus b^1$ and $(a \oplus b)^1 = a^1 * b^1$.

Or

(b) Show that

(i) Every chain is a distributive lattice

(ii) Every distributive lattice is modular.

20. (a) Describe

(i) Adjacency matrix and

(ii) Incidence matrix with their properties.

Or

(b) Prove that the following theorem : Set (T, V_0) be a rooted tree then prove that

(i) These are no cycles in T

(ii) V_0 is the only root of T and

(iii) Each vertex in T, other than V_0 has in degree one.