(7 pages) **Reg. No. :** ....

Code No. : 41384 E **Sub. Code : SMMA 41**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Fourth Semester

Mathematics – Main

ABSTRACT ALGEBRA – I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. Order of a non-zero element in $(Z,+)$ is ______________.

   (a) α (b) 0

   (c) 1 (d) 2

2. A group of order 12 cannot have a subgroup of order

   (a) 3 (b) 4

   (c) 5 (d) 6

3. If H and K are two finite subgroups of a group G the $[HK]=$

   (a) $\dfrac{|K|}{|H \cap K|}$ (b) $\dfrac{|H||K|}{|H \cap K|}$

   (c) $\dfrac{|H|}{|H \cap K|}$ (d) $\dfrac{|H|+|K|}{|H \cap K|}$

4. Let p be a prime number and a be any integer relatively prime to p. Then $a^{p-1} \equiv 1 \pmod p$

   (a) Lagrange's theorem

   (b) Fermat's theorem

   (c) Euler's theorem

   (d) Cauchy's theorem

5. The Kennel of a homomorphism $f : G \to G'$ is ______________.

   (a) a subgroup of G'

   (b) a normal subgroup of G'

   (c) a normal subgroup of G

   (d) $\{e\}$

6. $f:(R^*,\cdot)\to(R^+,\cdot)$ defined by $f(x)=|x|$ is

(a) one-one (b) homomorphism

(c) onto (d) (b) and (c)

7. The map $f:Z\to z$ defined by $f(x)=x^2+3$ is ————.

(a) a ring homomorphism

(b) not a ring homomorphism

(c) a ring-isomorphism

(d) a ring ephimorphism

8. An example of an infinite commutative ring without identity is ————.

(a) $(Z,+,\cdot)$ (b) $(Z_n,\oplus,\otimes)$

(c) $(2Z,+,\cdot)$ (d) $M_2(R)$

9. The product of the polynomials $2x+4$ and $4x^2+3x+1$ in $Z_6[x]$ is ————.

(a) $3x^3+2x^2+4x+4$

(b) $8x^2+2x^2+4x+4$

(c) $8x^3+22x^2+14x+4$

(d) $3x^3+2x^2+3x+4$

10. Let $f(x)$, $g(x)\in Z_4[x]$ be defined as $f(x)=x^2+2x+3$ and $g(x)=3x^2+2x$ then degree of $[f(x)+g(x)]=$ ————.

(a) 0 (b) 2

(c) 4 (d) 1

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Let G denote the set of all matrices of the form $\begin{pmatrix} x & x \\ x & x \end{pmatrix}$ where $x\in R$. Then prove that G is a group under matrix multiplication.

Or

(b) Prove that a non-empty subset H of a group G is a subgroup of G iff $a,b\in H\Rightarrow ab^{-1}\in H$.

12. (a) Let G be a group and $a,b\in G$ and then prove that

(i) order of a = order of a^{-1}

(ii) order of a = order of $b^{-1}ab$.

Or

(b) State and prove Fermat's theorem.

13. (a) Prove that any permutation can be expressed as a product of disjoint cycles.

Or

(b) $I(G)$ is a normal subgroup of $A(G)$ prove.

14. (a) Prove that the set of all real numbers of the form $a+b\sqrt{2}$ where $a,b \in Q$ under usual addition and multiplication is a ring.

Or

(b) Prove that a finite commutative ring R without zero-divisors is a field.

15. (a) Show that the homomorphic image of an internal domain need not be an integral domain.

Or

(b) Prove that $R[x]$ is an integral domain iff R is an integral domain.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Let H and K be two subgroups of a group G. Then prove that HK is a subgroup of G iff $HK = KH$.

Or

(b) If n is a prime number then prove that $Z_n - [0]$ is a group under multiplication modulo n.

17. (a) Prove that a subgroup of cyclic group is cyclic.

Or

(b) State and prove Lagrange's theorem.

18. (a) State and prove Cayley's theorem.

Or

(b) State and prove fundamental theorem of Homomorphism.

19. (a) Prove that any finite integral domain is a field.

Or

(b) Prove the characteristic of any field is either 0 or a prime number.

20. (a) Prove that any integral domain D can be embedded in a field F.

Or

(b) Let R be a ring and I be a subgroup of $(R, +)$. Prove that the multiplication in R/I given by $(I+a)(I+b) = I+ab$ is well defined if and only if I is an ideal of R.

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