(7 pages) **Reg. No. :** .....

Code No. : 7754 **Sub. Code : WMAE 11**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

First Semester

Mathematics – Elective – I

GRAPH THEORY AND APPLICATIONS

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — (15 × 1 = 15 marks)

Answer ALL questions.

Choose the correct answer:

1. Every Cubic graph has _____ number of vertices.

   (a) Odd (b) Even

   (c) Prime (d) 6

2. A clique of G is _____ graph of G

   (a) Subgraph

   (b) Complete subgraph

   (c) Induced subgraph with odd degrees

   (d) Induced subgraph with even degrees

3. An isolated vertex of a graph is a vertex with degree _____.

   (a) 1 (b) 2

   (c) 0 (d) 3

4. An edge $e = xy$ is a cut edge of a connected graph G if and only e belongs to

   (a) Cycle (b) No cycle

   (c) Path (d) None of the above

5. The connectivity and edge connectivity of a simple cubic graph G are _____.

   (a) Not equal (b) Greater than

   (c) Less than (d) Equal

6. Each block of G with at least three vertices is a _____ connected subgraph of G.

   (a) 1 (b) 2

   (c) 3 (d) 4

7. Every _____ graph contains a spanning tree.

   (a) Connected (b) Disconnected

   (c) Simple (d) All the above

Nesamony Memorial Christian College, Marthandam

8. If $\delta(G) \geq 2$, then G contains a _____.

(a) Path (b) Cycle

(c) Clique (d) Isolated vertex

9. If T is a tree with n vertices, then T contains _____ number of edges.

(a) n (b) $n-1$

(c) $n-2$ (d) $n-3$

10. A subset S of V is independent if and only if V/S is _____.

(a) Disconnected G (b) Covering of G

(c) Clique of G (d) None of the above

11. Every _____ has at most one perfect matching.

(a) K_n (b) C_n

(c) Tree (d) Simple graph

12. Every connected 3-regular graph having no cut edges has a

(a) 1-factor (b) 2-factor

(c) 3-factor (d) 4-factor

13. The Chromatic Number of C_6 _____.

(a) 1 (b) 2

(c) 3 (d) 4

14. Any Critical graph is _____.

(a) Disconnected

(b) Has an isolated vertex

(c) Connected

(d) Does not has an isolated vertex

15. Every k-critical graph is edge connected.

(a) k (b) $(k-1)$

(c) $(k-2)$ (d) $(k-3)$

PART B — ($5 \times 4 = 20$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that the number of edges of a simple graph of order n having ω component cannot exceed $\frac{(n-\omega)(n-\omega+1)}{2}$.

Or

(b) Prove that in a connected graph G with at least three vertices, any two longest paths have a vertex in common.

Nesamony Memorial Christian College, Marthandam

17. (a) Prove that a simple cubic connected graph G has a cut vertex if and only if it has a cut edge.

Or

(b) A connected graph G with at least two vertices contains at least two vertices that are not cut vertices.

18. (a) State and Prove Jordan's Theorem.

Or

(b) Prove that if e is not a loop of a connected graph G, then $\tau(G) = \tau(G-e) + \tau(G \circ e)$.

19. (a) Prove that a k-regular bipartite graph is 1-factorable.

Or

(b) Prove that every connected 3-regular graph having no cut edges has a 1- factor.

20. (a) State and prove Ore's Theorem.

Or

(b) Prove that in a critical graph G, no vertex cut is a clique.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

21. (a) Explain different Graph operations with an example.

Or

(b) Prove that if the simple graphs G_1 and G_2 are isomorphic, then $L(G_1)$ and $L(G_2)$ are isomorphic.

22. (a) State and Prove Whitney's theorem.

Or

(b) State and Prove Ear decomposition of a block.

23. (a) Prove that the number of edges in a tree on n vertices is $n-1$. Conversely a connected graph on n vertices and $n-1$ edges is a tree.

Or

(b) Prove that $\tau(K_n) = n^{n-2}$, where K_n is a complete graph on n vertices, $n \geq 2$.

Nesamony Memorial Christian College, Marthandam

24. (a) Prove that a bipartite graph G with bipartition (X,Y) and G has a matching that saturates all the vertices of X if and only if $|N(S)| \geq |S|$ for every subset S of X.

Or

(b) Prove that agraph G has a 1-factor if for all $O(G-S) \leq |S|$ and only if if $S \subseteq V$.

25. (a) Prove that if a connected graph G is neither an odd cycle nor a complete graph, then $\chi(G) \leq \Delta(G)$.

Or

(b) Prove that if for a simple 2-connected graph $G, \alpha \leq k$, then G is Hamiltonian.

Nesamony Memorial Christian College, Marthandam