Code No:SS5839

Sub Code: PMAM24

M.Sc. (CBCS) DEGREE SPECIAL SUPPLEMENTARY EXAMINATION,

APRIL 2020

SECOND SEMESTER

MATHEMATICS

DIFFERENTIAL GEOMETRY

(For those who joined in July 2017 onwards)

Time: Three hours

Maximum : 75 marks

Part A – (10X1=10 marks)

Answer ALL the questions, Choose the correct answer

1. A regular vector valued function r = R(u) of class m is called
   a) an analytic function b) a curve of class m
   c) a path of class m d) none
2. A point where $\dot{r} = 0$ is called a
   a) regular point b) reflexive point
   c) essential point d) singular point
3. A curve is a helix if and only if
   a) $\kappa = \tau$ b) $\kappa = 0$
   c) $\tau = 0$ d) $\kappa/\tau =$ constant
4. Radius of curvature ρ is
   a) κ b) $1/\kappa$
   c) τ d) $1/\tau$
5. ----------- point is defined as one for which $r_1 \times r_2 \neq 0$.
   a) ordinary b)singular
   c) variable d) irregular
6. The parametric curves are orthogonal if
   a) E = 0 b) F = 0
   c) G = 0 d) H = 0

P.T.O

7. Every helix on a --------------- is a geodesic

a) sphere

b) cone

c) helicoid

d) cylinder

8. Geodesics are -------------- of any particular parametric representation of the surface.

a) Dependent

b) independent

c) the curves

d) none

9. If at a point $LN - M^2 = 0$ then it is called a

------- point.

a)asymptotic

b) elliptic

c) hyperbolic

d) parabolic

10. The point where $L/E = M/F = N/G$ is called

----

a) the centre

b) the focus

c) an umbilic

d) the origin

PART B – (5 x 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b). Each answer should not exceed 250 words.

11.(a) Find the length of the circular helix

$R(u) = (a \cos u, a \sin u, bu), 0 < u < 2\pi$

Or

(b) Prove that a necessary and sufficient condition for a curve to be a straight line is $\kappa = 0$ at all points of the curve.

12.(a) Derive the equation of an involute of a curve C.

Or

(b) Calculate the curvature of the cubic curve given by

P.T.O

$r = (u, u^2, u^3)$.

13.(a) Show that on a right helicoid, the family of curves orthogonal to the curves $u \cos v$ = constant is the family $(u^2 + a^2)\sin^2 v$ = constant.

Or

(b) Find the coefficients of the direction which makes an angle $\pi/2$ with the direction whose coefficients are (l,m).

14.(a) Define orthogonal trajectory and find its differential equation.

Or

(b) A helicoids generated by the screw motion of a straight line which meets the axis at an angle α . Find the orthogonal trajectories of the generators.

15.(a) Show that the anchor ring contains all three types of points.

Or

(b) State and prove Euler's theorem on principal curvatures.

PART C – (5 x 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b). Each answer should not exceed 600 words.

16.(a) **Obtain** the curvature and torsion of a curve given as the **intersection** of two surfaces.

Or

(b) State and prove the Serret-Frenet formulae.

17.(a) Derive the equation of evolute

Or

(b) Prove that the product of curvatures at the corresponding points on a curve is equal to the product of torsions.

P.T.O

18.(a) Prove that, on the general surface, a necessary and sufficient condition that the curve $v = c$ be a geodesic is

$$EE_2 + FE_1 - 2EF_1 = 0$$

Or

(b) Prove that every helix on a cylinder is a geodesic.

19.(a) Derive the normal property of geodesics.

Or

(b) Derive the canonical geodesic equations.

20.(a) State and prove the Liouville's formula for K_g.

Or

(b) **Derive the Rodrigue's formula for the lines of curvature.**