(6 pages) **Reg. No. :** ....

Code No. : 30731 E **Sub. Code : EMMA 32**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2024.

Third Semester

Mathematics — Core

DIFFERENTIAL EQUATIONS AND APPLICATIONS

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. $y = vx$ is generally used for ———— differential equation.

   (a) Linear (b) Homogeneous

   (c) Bernoulli's (d) Cauchy's

2. The differential equation $(1+x)dy - ydx = 0$ has the general solution ————

   (a) $y = c(1-x)$ (b) $y = c + x$

   (c) $y = cx$ (d) $y = c + cx$

3. The solution of differential equation $(D^2 - 5D + 4)y = 0$ is ————

   (a) $y = c_1e^x + c_2e^{4x}$ (b) $y = c_1e^{-x} + c_2e^{4x}$

   (c) $y = c_1e^x + c_2e^{-4x}$ (d) $y = c_1e^{-x} + c^2e^{-4x}$

4. The solution of the differential equation $(D^2 + n^2)y = 0$ is ————

   (a) $y = A\cos nx + B\sin nx$

   (b) $y = Ae^{nx} + Be^{-nx}$

   (c) $y = (Ax + B)e^{nx}$

   (d) $y = (Ax + B)e^{-nx}$

5. If 'y' is the distance through which a body falls freely in time 't', its equation of motion is ————

   (a) $\frac{d^2y}{dt^2} = -g$ (b) $\frac{d^2y}{dt^2} = g$

   (c) $\frac{dy}{dt} = g$ (d) $\frac{dy}{dt} = -g$

6. The differential equation of the Brachistochrone problem is ————

   (a) $(1 + y'^2) = k$ (b) $y(1 + y'^2) = k$

   (c) $(1 + y')^2 - k$ (d) $y(1 + y')^2 = k$

7. The partial differential equation of $z = (x+a)^2 + (y+b)^2 + c^2$ by eliminating the arbitrary constants 'a' and 'b' is ————

(a) $z = p^2 + q^2 + 4c^2$ (b) $z = p^2 + q^2 + c^2$

(c) $2z = 2p^2 + q^2 + c^2$ (d) $4z = p^2 + q^2 + 4c^2$

8. The partial differential equation of by eliminating the arbitrary constants 'a' and 'b' is ————

(a) $py - qx = 0$ (b) $px - qy = 0$

(c) $px + qy = 0$ (d) $py + qx = 0$

9. The general solution of $2p + 3q = 1$ is ————

(a) $\varphi(2x+3y,\ y-3z) = 0$

(b) $\varphi(2x+3y, y+3z) = 0$

(c) $\varphi(3x-2y,\ y-3z) = 0$

(d) $\varphi(2x-3y,\ y-3z) = 0$

10. The solution of $p = \tan(px - y)$ is ————

(a) $y = cx - \tan^{-1} c$ (b) $y = cx + \tan c$

(c) $x = cy + \tan y$ (d) $x = cy + \tan x$

PART B — (5 × 5 = 25 marks)

Answer ALL questions by choosing either (a) or (b).
Each answer should not exceed 250 words.

11. (a) Solve $y - x\dfrac{dy}{dx} = a(y^2 + dy/dx)$.

Or

(b) Solve $\dfrac{dy}{dx} + y\cos x = \dfrac{1}{2}\sin 2x$.

12. (a) Solve $p^2 + (x + y - 2y/x)p + xy + y^2 / x^2 - y - y^2 / x = 0$.

Or

(b) Solve $(D^3 - D^2 - D + 1)y = 1 + x^2$.

13. (a) Solve $x^2 \dfrac{d^2y}{dx^2} - (x^2 + 2x)\dfrac{dy}{dx} + (x+2)y = x^3 e^x$.

Or

(b) Solve the equation $L\dfrac{dI}{dt} + RI = E$ under the initial conditions $I = I_0$, $E = E_0 e^{-kt}$ at $t = 0$.

14. (a) Eliminate the arbitrary function f from $f(x^2+y^2+z^2, z^2-2xy)=0$.

Or

(b) Solve $(y+z)p+(z+x)q=x+y$.

15. (a) Prove that the characteristics of $q=3p^2$ pass through the point (–1, 0, 0) generate the cone $(x+1)^2+12yz=0$.

Or

(b) Solve $p^2+q^2=z^2(x^2+y^2)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions by choosing either (a) or (b).
Each answer should not exceed 600 words.

16. (a) Solve $\frac{dy}{dx}+\frac{10x+8y-12}{7x+5y-9}=0$.

Or

(b) Solve $(y^2+2x^2y)dx+(2x^3-xy)dy=0$.

17. (a) Solve $(D^4+D^3+D^2)y=5x^2+\cos x$.

Or

(b) Solve $(D^2-2d+4)y=e^x\sin x$.

18. (a) Solve $4x^2\frac{d^2y}{dx^2}+4x^5\frac{dy}{dx}+(x^8+6x^4+4)y=0$.

Or

(b) Find the time required to empty a cylindrical tank 1 metre in diameter and 4 metres long through the hole 5 cm diameter if the tank is initially full and its axis is (a) vertical.

19. (a) Solve $px(y^2+z)-qy(x^2+z)=z(x^2-y^2)$. Find the surface that contains the straight lines $x+y=0, z=1$.

Or

(b) Determine the surface which satisfies the differential equation

$$(x^2-a^2)p+(xy-az\tan\alpha)q=xz-ay\cot\alpha$$

and passes through the curve $x^2+y^2=a^2$, $z=0$.

20. (a) Solve (i) $q=xp+p^2$ and (ii) $p=y^2q^2$.

Or

(b) Solve $p(1+q^2)=q(z-1)$.