(6 pages) **Reg. No. :** ....

Code No. : 7072 Sub. Code : C 44 M

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2013.

Fourth Semester

Mathematics

CALCULUS OF VARIATIONS AND INTEGRAL EQUATIONS

(For those who joined in July 2008–2011)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

1. State the necessary condition for the function $z = f(x, y)$ of two independent variables x and y to prove a relative maximum at an interior point (x_0, y_0) of the region of definition.

2. What is the base problem in calculus of variation?

3. Write down the functional to be maximized and the constraints for determining the curve of length l which passes through the point $(0, 0)$ and (1, 0) and for which the area between the curve at the X-axis is a maximum.

4. What do you mean by variational problem with variable end point?

5. Define a Fredholm integral equation.

6. Define Green's function.

7. Give an example for separable Kernel.

8. Define the term characteristic value of an integral equation.

9. State the condition for the existence of continuous solution to the Fredholm equation of the first kind.

10. Give an example of an integral equation of the Volterra type.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the minimal surface of revolution passing through two given points.

Or

(b) Determine the stationary function $y(x)$ for the problem, $\delta\left\{\int_0^1 y'^2\,dx + [y(1)]^2\right\} = 0;\ y(0) = 1$.

12. (a) Use the calculus of variation of find the shortest distance between the line $y = x$ and the parabola $y^2 = x - 1$.

Or

(b) Find a Euler's equation relevant to the problem $\delta\lambda = 0$, where

$$\lambda = \frac{\int_{x_1}^{x_2}\left[sy''^2 - py'^2 + qy^2\right]dx}{\int_{x_1}^{x_2} ry^2 dx}.$$

13. (a) Find the boundary value problem equivalent to the integral equation

$$y(x) = \lambda \int_0^x \frac{\xi}{l}(l - x)\, y(\xi) d\xi + \lambda \int_l^x \frac{x}{l}(x - \xi) y(\xi)\, d\xi.$$

Or

(b) Show that if $y(x)$ satisfies the differential equation, $\frac{d^2y}{dx^2} + xy = 1$ and the condition $y(0) = y'(0) = 0$; then y also satisfies the Volterra equation

$$y(x) = \int_0^{x_1} \xi(\xi - x) y(\xi)\, d\xi + \frac{1}{2}x^2.$$

14. (a) Find the characteristic values of λ for the equation $y(x) = \lambda \int_0^{2\pi} \sin(x + \xi) y(\xi) d\xi$

Or

(b) Obtain an approximate solution of the integral equation $y(x) = \int_0^1 \sin(x\xi) y(\xi) d\xi + x^2$ by replacing $\sin(x\xi)$ by the first term of its power series expansion.

15. (a) Apply iterative method and obtain the results of two successive substitution to the equation $y(x) = \int_0^x (x + \xi)\, y(\xi)\, d\xi + 1$ taking $y^0(x) = 1$.

Or

(b) Show that the iterative procedure will converge when $|\lambda| < 3$ for the equation $y(x) = \lambda \int_0^1 x\xi y(\xi) d\xi + 1$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Determine the stationary function associated with the integral $I = \int_0^1 (Ty'^2 - \rho\,\omega^2 y^2)\, dx$ where T, ρ and ω are given constants or functions of x.

Or

(b) Derive the Euler equation of the problem $\int_{x_1}^{x_2}\left[a(x)y''^2 - b(x)y'^2 + c(x)y^2\right]dx = 0$; obtain the associated natural boundary condition.

17. (a) Define the term generalized coordinates, generalized velocity and generalized force. Derive the Lagrange's equation for the compound pendulum.

Or

(b) A particle of mass 'm' is falling vertically under the action of gravity and its motion is prescribed by a force numerically equal to a constant c times its velocity 'x'. Describe the form of Hamilton's principle and obtain the Euler equation.

18. (a) Obtain the Green's function for the Bessel operator of order n, $Ly = \frac{d}{dx}\left(x\frac{dy}{dx}\right) - \frac{n^2}{x}y$ relevant to the end condition $y(0) = y(1) = 0$.

Or

(b) Reduce the problem

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + \left(\lambda x^2 - n^2\right)y = 0, \quad y(0) = y(1) = 0$$

to an integral equation when $n \neq 0$.

19. (a) Solve the equation $y(x) = 1 + \lambda\int_{-\pi}^{\pi} e^{iw(x-\xi)}y(\xi)d\xi$ considering separately all exceptional cases.

Or

(b) Solve $y(x) = \lambda\int_0^1 (1 - 3x\xi)y(\xi)d\xi + F(x)$ by reducing it to a system of linear equation.

20. (a) Obtain the exact solution of the equation $y(x) = \sin x + \lambda\int_0^{2\pi}\cos(x+\xi)\,y(\xi)\,d\xi$.

Or

(b) Obtain the resolvant kernal associated with $k(x,\xi) = e^{-(x-\xi)}$ in the interval $(0, a)$.
