(7 pages) **Reg. No. :** ....

Code No. : 6437 **Sub. Code : HCSM 11**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2014.

First Semester

Computer Science

MATHEMATICAL FOUNDATION FOR COMPUTER SCIENCE

(For those who joined in July 2012 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL the questions.

Choose the correct answer.

1. A compound proposition that is always true regardless of the truth values of the propositions in it is called ————.

   (a) Contradiction (b) Tautology

   (c) Normal form (d) None

2. A statement formula which is equivalent to the given formula and expressed as sum of minterms is called ————.

   (a) PCNF (b) CNF

   (c) PDNF (d) DNF

3. A function $f : A \rightarrow B$ which is both one-one and onto is called ————.

   (a) injection (b) surjection

   (c) bijection (d) none

4. A reflection R, defined on a set A, which is reflexive, symmetric and transitive is called ————.

   (a) Partial ordering relation

   (b) Total ordering relation

   (c) Equivalence relation

   (d) None

5. For any commutative monoid $(M, *)$, the set of idempotent elements of M form a ————.

   (a) sub monoid (b) subgroup

   (c) group (d) none

6. Let H be a subgroup of a group G then Lagrange's theorem states that ————.

(a) $O(G) \mid O(H)$ (b) $O(H) \mid O(G)$

(c) $O(H) \times O(G)$ (d) $O(G) \times O(H)$

7. A graph with no edges is called ————.

(a) Connected graph (b) Null graph

(c) Euler graph (d) None

8. In a simple graph with n vertices, the length of any elementary path is less than (or) equal to ————.

(a) n (b) $n+1$

(c) $n-1$ (d) $2n$

9. The number of labeled trees with n vertices $(n \geq 2)$ is ————.

(a) n^2 (b) n^{n-1}

(c) n^{n-2} (d) $n+1$

10. The largest level of the vertices of a tree is called ———— of a tree.

(a) weight (b) height

(c) diameter (d) radius

PART B — (5 × 5 = 25 marks)

Answer ALL the questions, choosing either (a) or (b), each answer should not exceed 250 words.

11. (a) Construct the truth table for $\neg(P \to Q) \to P$.

Or

(b) Obtain the DNF and CNF for $(P \to (Q \wedge R)) \wedge (\neg P \to (\neg Q \wedge \neg R))$.

12. (a) (i) Define partially ordered set. Give example.

(ii) Give an example of a relation which is symmetric, transitive but not reflexive on $\{a, b, c\}$.

Or

(b) Let R be a relation on a set A. Then define $R = \{(a, b) \in A \times A / (b, a) \in R\}$. Prove that if (A, R) is poset then (A, R^{-1}) is also a poset.

13. (a) Show that in a group $(G, *)$ if for any $a, b \in G$, $(a * b)^2 = a^2 * b^2$ then $(G, *)$ must be abelian.

Or

(b) Let S be a non empty set and $P(S)$ denote the power set of S. Verify whether $(P(S), \cap)$ is a group.

14. (a) Prove that the number of vertices of odd degree in a graph is always even.

Or

(b) Prove that if a graph G has not more than two vertices of odd degree, then there can be Euler path in G.

15. (a) Prove that a graph G with n vertices, $n-1$ edges and no circuit is connected.

Or

(b) Explain Kruskal's algorithm to find the shortest spanning tree with an example.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL the questions, choosing either (a) or (b), each answer should not exceed 600 words.

16. (a) Obtain the PCNF and PDNF of the statement $(Q \vee (P \wedge R)) \wedge \neg((P \vee R) \wedge Q)$.

Or

(b) Show that $R \to S$ can be derived from the premises $P \to (Q \to S)$, $\neg R \vee P$ and Q.

17. (a) Let A, B, C be any three non-empty sets. Let $f: A \to B$ and $g: B \to C$ be mappings. If f and g are onto. Prove that $g \circ f: A \to C$ is onto. Also give an example to show that $g \circ f$ may be onto but both f and g need not be onto.

Or

(b) A function $f: R \to R$ is defined by $f(x) = \frac{ax^2 + 6x - 8}{a + 6x - 8x^2}$. Find the integral values of 'a' for which f is onto. Is the function one to one for $a = 3$? Justify your claim.

18. (a) Prove that every finite group of order n is isomorphic to a permutation group of order n.

Or

(b) Let $(G, *)$ be a group and $a \in G$. Let $f: G \to G$ be given by $f(x) = a * x * a^{-1}$ for every $x \in G$, prove that f is an isomorphism G onto G.

19. (a) A connected graph G is an Euler path iff it can be decomposed into circuits.

Or

(b) Prove that in a complete graph with n vertices there are $\frac{(n-1)}{2}$ edge-disjoint Hamiltonian circuits if n is odd and $n \geq 3$.

20. (a) Prove that a tree with n vertices has $n-1$ edges.

Or

(b) Write the adjacency matrix of the digraph :

$G = \{(V_1, V_3), (V_1, V_2), (V_2, V_4), (V_3, V_1),$

$(V_2, V_3), (V_3, V_4), (V_4, V_1), (V_4, V_2), (V_4, V_3)\}$.

Also draw the graph.

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