

17. (a) Prove that any linear transformation can be expressed as a product of translation, rotation, magnification or contraction and inversion.

Or

- (b) Find all circles which are orthogonal to $|z| = 1$ and $|z - 1| = 4$.

18. (a) State and prove Cauchy's theorem for a rectangle.

Or

- (b) State and prove Taylor's theorem.

19. (a) State and prove the generalized version of argument principle.

Or

- (b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$.

20. (a) Derive the Schwarz formula for harmonic functions.

Or

- (b) If $f(z)$ is analytic in the region Ω , containing z_0 , then prove that the representation

$$f(z) = f(z_0) + \frac{f'(z_0)(z - z_0)}{1!} + \dots + \frac{f^n(z_0)(z - z_0)^n}{n!} + \dots$$

is valid in the largest open disk of center z_0 contained in Ω .

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M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2012.

Fourth Semester

Mathematics

COMPLEX ANALYSIS

(For those who joined in July 2008 and afterwards)

Time : Three hours

Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL the questions.

1. Write down the Cauchy-Riemann differential equations.
2. Prove that a convergent sequence is bounded.
3. Prove that the reflection $z \rightarrow \bar{z}$ is not a linear transformation.
4. When do we say that a linear transformation is parabolic?
5. If C is the circle $|z| = 1$, prove that $\int_C \frac{z}{z-2} dz = 0$.

6. Define a meromorphic function.
7. What do you mean by simply connected region?
8. Define residues.
9. Show that $\log r$ is a harmonic function.
10. When the limit function of a sequence of analytic functions is analytic?

PART B — (5 × 5 = 25 marks)

Answer ALL the questions choosing either (a) or (b).

11. (a) If a function is analytic, show that it is independent of $\bar{z}$.
- Or
- (b) Determine the single valued analytic branch of the inverse cosine function $\arccos(z)$.
12. (a) Show that the cross ratio is real if and only if the four points lie on a circle or a straight line.

Or

- (b) Show that any linear transformation having two finite fixed points a, b can be written in the form $\frac{w-a}{w-b} = k \frac{z-a}{z-b}$.

13. (a) State and prove the necessary and sufficient condition for a line integral depends only on end points.

Or

- (b) Prove that a non-constant analytic function maps open sets onto open sets.

14. (a) Show that single valued analytic branch of $\log z$ can be defined in any simply connected region which does not contain the origin.

Or

- (b) State and prove Rouches's theorem.

15. (a) State and prove the maximum principle for harmonic functions.

Or

- (b) What is the coefficient of z^7 in the Taylor's development of $\tan z$?

PART C — (5 × 8 = 40 marks)

Answer ALL the questions choosing either (a) or (b).

16. (a) Prove that every rational function has a representation by partial fractions.

Or

- (b) Use the addition formulas to separate $\cos(x+iy)$, $\sin(x+iy)$ in real and imaginary parts.