(7 pages) **Reg. No. :**..

Code No. : 20580 E **Sub. Code : SMMA 63**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2021.

Sixth Semester

Mathematics — Core

GRAPH THEORY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. In any group of more than one people, the number of people having the same number of friends inside the group is

   (a) 3 9 (b) 2

   (c) 4 (d) 5

2. For a graph with 6 points, the independence number is 2. Then the covering number is ___________.

   (a) 4 (b) 2

   (c) 6 (d) 3

3. Girth of C_8 is

(a) 2 (b) 4

(c) 6 (d) 8

4. If G is a connected graph then $w(G)$ = ____________.

(a) 0 (b) 2

(c) 1 (d) number of edges

5. Number of edges of a tree of order 10 is

(a) 10 (b) 11

(c) 9 (d) 5

6. Which theorem is stronger than Dirac's theorem?

(a) Cayley theorem

(b) Euler's theorem

(c) Hamilton's theorem

(d) Chvatal's theorem

7. In any connected plane (p, q) graph with r faces, the minimum number of edges is

(a) $\frac{3r}{2}$ (b) $\frac{2r}{3}$

(c) $3p+6$ (d) $p-1$

8. The chromatic number of a tree T with atleast 2 points is

(a) 1 (b) 2

(c) 0 (d) 3

9. If G is a (p, q) graph and $f(G, \lambda) = \lambda^r + s\lambda^{r-1} + \ldots$ then r, s are respectively

(a) p, q (b) q, p

(c) $q, -p$ (d) $p, -q$

10. In a digraph,

(a) $\Sigma d^+(v) = \Sigma d^-(v) = q$ (b) $\Sigma d^+(v) = 2q$

(c) $\Sigma d^-(v) = 2q$ (d) None of these

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that isomorphism preserves the degree of vertices.

Or

(b) Prove that every graph is an intersection graph.

12. (a) Show that the partition $P = (6, 6, 5, 4, 3, 3, 1)$ is not graphical.

Or

(b) Show that any $u - v$ walk contains a $u - v$ path.

13. (a) If G is a graph in which the degree of every vertex is atleast two then show that G contains a cycle.

Or

(b) Prove that in a tree, between any two points there is a unique path.

14. (a) Prove that every polyhedron has atleast two faces with the same number of edges on the boundary.

Or

(b) State and prove Euler's polyhedron formula.

15. (a) Define :

(i) Directed walk

(ii) Degree pair

(iii) Digraph.

.Or

(b) Show that $\lambda^4 - 3\lambda^3 + 3\lambda^2$ cannot be the chromatic polynomial of any graph.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If G_1 is a (p_1, q_1) and G_2 is a (p_2, q_2) graph then prove that $G_1 + G_2$ is a $(p_1 + p_2, q_1 + q_2 + p_1 p_2)$ graph and $G_1 \times G_2$ is a $(p_1 \cdot p_2, q_1 p_2 + q_2 p_1)$

Or

(b) Prove that the maximum number of lines among all p-point graphs with no triangles is $\left[\frac{p^2}{4}\right]$.

17. (a) Prove that a partition $P = (d_1, d_2, ..., d_p)$ of even number into p parts with $p-1 \geq d_1 \geq d_2 \geq .... \geq d_p$ is graphical iff the modified partition $P^1 = \left(d_2 - 1, d_3 - 1, ... \frac{d-1}{d_1+1}, .\; ..., d_p\right)$ is graphical.

Or

(b) Prove that a graph G with atleast two points is bipartite iff all its cycles are of even length.

18. (a) Show that the Petersen graph is non-hamiltonian.

Or

(b) Prove that the following statements are equivalent for a connected graph G.

(i) G is Eulerian.

(ii) Every point of G has even degree.

(iii) The set of edges of G can be partitioned into cycles.

19. (a) (i) Prove that the graphs K_5 and $K_{3,3}$ are not planar.

(ii) If G is a plane connected (p, q) graph without triangles and $p \geq 3$ then prove that $q \leq 2p - 4$.

Or

(b) Prove that $\chi'(K_n) = \begin{cases} n & \text{if } n \text{ is odd } (n \neq 1) \\ n-1 & \text{if } n \text{ is even} \end{cases}$.

20. (a) Prove that the coefficients of $f(G, \lambda)$ are alternate in sign. Also prove that if G is a (p, q) graph then the coefficient of λ^{p-1} is $-q$.

Or

(b) Prove that a weak diagraph D is Eulerian iff every point of D has equal in-degree and out-degree.