(8 pages)

**Reg. No. :** ....................................

**Code No. : 7771** **Sub. Code : WMAM 33**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2024.

Third Semester

Mathematics — Core

TOPOLOGY

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — (15 × 1 = 15 marks)

Answer ALL questions.

Choose the correct answer :

1. Which one of the following is a topology on $X = \{a,b,c,d\}$

   (a) $\{\phi, X, \{a,b\}, \{b,c\}, \{a,b,c\}\}$

   (b) $\{\phi, X, \{a\}, \{b\}, \{a\,b\}, \{a,c\}\}$

   (c) $\{\phi, X, \{a,b,c\}, \{b,c,a\}\}$

   (d) $\{\phi, X, \{a\}, \{a,b\}, \{a,b,c\}\}$

2. I. In $Y = [0, 1]$, its subspace topology and its order topology are the same

   II. In $Y = [0,1) \cup \{2\}$, its subspace topology and its order topology are the same

   (a) Both I and II are true

   (b) I is true but II is not true

   (c) I is not true but II is true

   (d) Neither I nor II is true

3. Let $X = \{a,b,c\}$, $\tau = \{\phi, X, \{a,b\}, \{b,c\}, \{cb\}$. Let $x_n = b$ for all $n$. The sequence $\{x_n\}$ converges to

   (a) $b$ only (b) $c$ only

   (c) $a$ only (d) $a$, $b$ and $c$

4. Let $f : A \to B$. Let $A_0 \subset A$ and $B_0 \subset B$, then the false statement is

   (a) $A_0 \subset f^{-1}(f(A_0))$

   (b) $f(F^{-1}(B_0)) \subset B_0$

   (c) $B_0 \subset B_1 \Rightarrow f^{-1}(B_0) \subset f^{-1}(B_1)$

   (d) $f(A_0 \cap A_1) = f(A_0) \cap f(A_1)$

5. Which one of the following is not true for box and product topologies?

(a) the box topology is finer than the product topology

(b) for finite products, two topologies are precisely the same

(c) the product topology is finer than the box topology

(d) for infinite products, two topologies are different

6. The interval (2, 6) is

(a) $B(2,4)$ (b) $B(3,2)$

(c) $B(2-\varepsilon, 6-\varepsilon)$ (d) $B(4,2)$

7. The only connected subspace of $Q$ are

(a) the subsets of $Q$

(b) the one-point sets

(c) $\phi$ and $R$

(d) the open intervals in R

8. Consider

I $R$ is connected.

II $R^\omega$ is connected in the box topology

(a) Both I and II are true

(b) I is true but II is not true

(c) II is true but I is not true

(d) Neither I nor II is time



9. The set (0, 5) in $R$ is

(a) both connected and locally connected

(b) connected but not locally connected

(c) locally connected but not connected

(d) neither connected nor locally connected

10. Which one of the following is not compact

(a) Space $X$ containing 5 points

(b) The subspace $X = \{0\} \cup \{1/n \in n/Z_+\}$

(c) $R$

(d) [0, 1]

11. A space $X$ said to be limit point compact if

(a) $X$ has a limit point

(b) every subset of $X$ has a limit point

(c) every infinite subset of $X$ has a limit point

(d) every finite subset of $X$ has a limit point

12. Consider

I $Q$ is locally compact

II $R^\omega$ is locally compact

(a) Both I and II are true

(b) I is true but II is not true

(c) II is true but I is not true

(d) Neither I nor II is true

13. Which one of the following is false

(a) A subspace of a topological space is a topology

(b) A subspace of a Hausdorff space is Hausdorff

(c) A subspace of a regular space is regular

(d) A subspace of a normal space is normal

14. Which one of the following is true

(a) $R_l^2$ is not normal

(b) $R_l$ is not regular

(c) $R_l^2$ is not regular

(d) $R_l$ is not normal

15. Complete regularity lies between

(a) Hausdorff and regularity

(b) Regularity and completeness

(c) Completeness and normality

(d) Regularity and normality

PART B — (5 × 4 = 20 marks)

Answer ALL questions, choosing either (a) or (b).
Each answer should not exceed 250 words.

16. (a) State and prove a criterion in terms of the bases for determining whether one topology is finer than another.

Or

(b) Let $Y$ be a subspace of $X$; let $A$ be a subset of $Y$. Express the closure of $A$ in $Y$ in terms of $\overline{A}$.



17. (a) Let $f: A \to XxY$ be given by the equation $f(a) = (f_1(a), f_2(a))$. Prove that $f$ is continuous if and only if the functions $f_1 : A \to x$ and $f_2 : A \to Y$ are continuous.

Or

(b) State and prove the sequence lemma.

18. (a) Define a connected space. Prove that the rationals $Q$ are not connected.

Or

(b) If $X$ is a topological space, prove that each path component of $X$ lies in a component of $X$. If $X$ is locally path connected, prove that the components and the path components of $X$ are the same.

19. (a) Prove that every closed subspace of a compact space is compact.

Or

(b) Let $X$ be a locally compact Hausdroff space. Let $A$ be a subspace of $X$. If $A$ is closed in $X$ or open in $X$, prove that $A$ is locally compact.

20. (a) Show that every compact Hausdroff space is normal.

Or

(b) Prove that a product of completely regular spaces is completely regular.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b)
Each answer should not exceed 600 words.

21. (a) Define any three topologies on the real line $R$ and bring out the relation between these topologies.

Or

(b) Define the closure and the limit points of a set bring out a relationship between the closure of a set and the limit points of a set.

22. (a) Let $X$ and $Y$ be topological spaces, let $f: X \to Y$ prove that the following are equivalent.

(i) $f$ is continuous

(ii) for every subset $A$ of $X$, one has $f(\overline{A}) \subseteq \overline{f(A)}$

(iii) for every closed set $B$ of $Y$, the set $f^{-1}(B)$ is closed in $X$.

Or

(b) Prove that the function $D(x, y) = \sup\left\{\frac{J(x_i, y_i)}{i}\right\}$ is a metric and that induces the product topology on $R^w$.



23. (a) Prove that a finite Cartesian product of connected spaces is connected.

Or

(b) Given $X$, an d $x, y \in X$, define $x \sim y$ is there is a connected subspace of $X$ containing both $x$ and $y$ show that $\sim$ is an equivalence relation and hence define components. What are the properties of components? Explain.

24. (a) State and prove the tube lemma.

Or

(b) Let $X$ be a metrizable space. Prove that sequentially compactness implies compactness.

25. (a) Prove that every regular space with a countable basis is normal.

Or

(b) State and prove the Urysohn lemma.

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