(6 pages) **Reg. No. :** ....

Code No. : 6370 **Sub. Code : HMAE 41**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2016.

Fourth Semester

Mathematics

Elective — GRAPH THEORY

(For those who joined in July 2012 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If H is a spanning subgraph of a graph G, then G and H have the same number of

   (a) vertices

   (b) edges

   (c) vertices and edges

   (d) none of these

2. If G is a graph with υ vertices and $\in$ edges, then the adjacency matrix of G is

   (a) $\upsilon \times \upsilon$ matrix (b) $(\in \times \in)$ matrix

   (c) $(\upsilon \times \in)$ matrix (d) none of these

3. The vertex connectivity of a tree with υ vertices is

   (a) 1 (b) υ

   (c) $\upsilon - 1$ (d) none of these

4. The complete bipartite graph $K_{2,3}$ is

   (a) Hamiltonian

   (b) Eulerian

   (c) Both Hamiltonian and Eulerian

   (d) None of these

5. A complete graph on odd number of vertices has __________.

   (a) perfect matching (b) maximum matching

   (c) no matching (d) none of these

6. The edge chromatic number of an even cycle is

   (a) 1 (b) 2

   (c) 3 (d) none of these

7. The covering number of a cycle on $3n$ vertices is

(a) 3 (b) n

(c) $3n$ (d) none of these

8. The value of $r(3, 4)$ is

(a) 3 (b) 4

(c) 9 (d) none of these

9. The vertex chromatic number a complete graph on n vertices is

(a) n (b) $n-1$

(c) 1 (d) none of these

10. The vertex chromatic number of a complete bipartite graph $K_{m,n}$ is

(a) 2 (b) m

(c) n (d) none of these

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that a connected graph is a tree if and only if every edge is a cut edge.

Or

(b) If G is loopless graph, prove that G is a tree if and only if any two vertices are connected by a unique path.

12. (a) Define vertex connectivity, edge connectivity of a graph with example. Give an example of a graph in which $K < K' < \delta$.

Or

(b) If a graph G has at most two vertices of odd degree, then prove that G has an Euler trail.

13. (a) Prove that, in a bipartite graph, the number of edges in a maximum matching is equal to the number of vertices in a minimum covering.

Or

(b) Let G be a connected graph that is not an odd cycle. Then prove that G has a 2-edge colouring in which both colours are represented at each vertex of degree atleast two.

14. (a) If $\delta' > 0$, prove that $\alpha' + \beta' = v$.

Or

(b) Prove that $r(3, 3) = 6$.

15. (a) Define K-critical graph. Prove that every K-chromatic graph has at least K vertices of degree atleast $K-1$.

Or

(b) Prove that for any graph G, $\pi_k(G)$ is a polynomial in k of degree υ, with integer coefficients, leading term k^{υ} and constant term zero. Further more, prove that, the coefficient of $\pi_k(G)$ is alternate in sign.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Prove that a graph is bipartite if and only if it contains no odd cycle.

Or

(b) Prove that $\tau(k_n) = n^{n-2}$.

17. (a) Prove that a graph with $\upsilon \geq 3$ is 2-connected if and only if any two vertices of G are connected by at least two internally-disjoint paths.

Or

(b) Let G be a simple graph with degree sequence $(d_1, d_2, ..., d_{\upsilon})$, where $d_1 \leq d_2 \leq ... \leq d_{\upsilon}$ and $\upsilon \geq 3$. Suppose that there is no value of m less than $\upsilon/2$ for which $d_m \leq m$ and $d_{\upsilon-m} < \upsilon - m$. Prove that G is Hamiltonian.

18. (a) Let G be a bipartite graph with bipartition (X, Y). Prove that G contains a matching that saturates every vertex of X if and only if $|N(S)| \geq |S|$ for all $S \subseteq X$.

Or

(b) If G is simple, then prove that either $\chi' = \Delta$ or $\chi' = \Delta + 1$.

19. (a) Prove that for any two integers $k \geq 2$ and $l \geq 2$, $r(k, l) \leq r(k, l-1) + r(k-1, l)$.

Or

(b) Prove that $r(k, k) \geq 2^{k/2}$.

20. (a) If G is a connected simple graph and is neither an odd cycle nor a complete graph, prove that $\chi \leq \Delta$.

Or

(b) Prove that for any positive integer k, there exists a k-chromatic graph containing no triangle.

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