(6 pages) **Reg. No. :** ....

Code No. : 7124 **Sub. Code : PMAE 23**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Second Semester

Mathematics

Elective — PARTIAL DIFFERENTIAL EQUATIONS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The equation $\sum_{i=1}^{n} F_i dx_i = 0$ is called a ———— differential equation.
   (a) linear (b) non- linear
   (c) pfaffian (d) partial

2. A necessary and sufficient condition that there exists between two functions $u(x, y)$ and $v(x, y)$ a relation $F(u, v) = 0$, not involving x or y explicitly is ————.
   (a) $\frac{\partial(u, v)}{\partial(x, y)} = 0$ (b) $\partial(x, y) = 0$
   (c) $\partial(u, v) = 0$ (d) $\frac{\partial(x, y)}{\partial(u, v)} = 0$

3. A ———— differential equation is one in which at least two independent variables enter.
   (a) Ordinary (b) Partial
   (c) Linear (d) Non - linear

4. $\frac{\partial\theta}{\partial x} + \frac{\partial\theta}{\partial y} = 0$ is a ———— order equation.
   (a) zero (b) first
   (c) second (d) third

5. Two first order PDE's are said to be ———— if they have a common solution
   (a) compatible (b) partial
   (c) linear (d) non — linear

6. The first order partial differential equation $f(x, y, z, p, q) = 0$ is also a solution of the equation $g(x, y, z, p, q) = 0$. These equations are said to be ————.
   (a) compatible (b) linear
   (c) non- linear (d) partial

7. The equation $\frac{\partial^2\theta}{\partial x^2} = \frac{1}{k}\frac{\partial\theta}{\partial t}$ is called ———— equation.
   (a) Poisson's (b) Laplace's
   (c) Telegraphy (d) Radio

8. The equation $\nabla^2\varphi = 0$ is known as ———— equation.

(a) Poisson's (b) Laplace's

(c) Telegraphy (d) Radio

9. The planes perpendicular to the normals envelope a cone. This cone is called the ———— cone through the point.

(a) characteristic (b) enveloping

(c) surface (d) point

10. If φ is Positive definite the characteristic cones and conoids are imaginary then the equation is ———— at P.

(a) parabolic (b) elliptic

(c) hyperbolic (d) zero

SECTION B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the integral Curves of the equation

$$\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}.$$

Or

(b) If X is a vector such that X_0 *curl* $X = 0$ and μ is an arbitrary function x, y, z then prove that $(\mu X)_0$ *curl* $(\mu X) = 0$.

12. (a) Eliminate the constants and α and β from the equation, $(x-\alpha)^2 + (y-\beta)^2 = r^2$.

Or

(b) Eliminate the arbitrary function f from the equation, $Z = xy + f(x^2 + y^2)$.

13. (a) Prove that along every characteristic strip of the equation $F(x, y, z, p, q) = 0$ the function $F(x, y, z, p, q)$ is a constant.

Or

(b) Find the complete integral of the equation $\sqrt{p} + \sqrt{q} = 1$.

14. (a) If u is the complement function and z_1 a particular integral of a linear partial differential equation, then prove that $u + z_1$ is a general solution of the equation.

Or

(b) Find a particular integral of the equation $(D^2 - D')Z = 2y - x^2$.

15. (a) Classify the equation, $u_{xx} + x^2 u_{yy} = 0$.

Or

(b) Classify the equation, $(1+x^2)u_{xx} + (1+y^2)u_{yy} + xu_x + yu_y = 0$.

SECTION C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Solve the equation, $\frac{dx}{y+\alpha z} = \frac{dy}{z+\beta x} = \frac{dz}{x+\gamma y}$.

Or

(b) Verify that the differential equation : $(y^2+yz)dx+(xz+z^2)dy+(y^2-xy)dz=0$ is integrable and find its primitive.

17. (a) Find the general solution of the differential equation, $x^2\frac{\partial z}{\partial x}+y^2\frac{\partial z}{\partial y}=(x+y)z$.

Or

(b) Find the general integral of the partial differential equation,

$y^2p-xyq=x(x-2y)$.

18. (a) Prove that if a characteristic strip contains at least one integral element of $F(x,y,z,p,q)=0$ it is an integral strip of the equation $F(x,y,z,z_x,z_y)=0$.

Or

(b) Show that the equations $xp=yq, z(xp+yp)=2xy$ are compatible.

19. (a) Find the solution of the equation,

$$\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}=x-y.$$

Or

(b) Find a particular integral of the equation :

$(D^2-D')Z=e^{2x+y}$.

20. (a) By separating the variables, determine the solution for the one – dimensional diffusion equation.

Or

(b) Solve $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=\frac{1}{k}\frac{\partial z}{\partial t}$, by separating the variables.