(7 pages) **Reg. No. :** ....

Code No. : 7365 **Sub. Code : HMAM 33**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2015.

Third Semester

Mathematics

DIFFERENTIAL GEOMETRY

(For those who joined in July 2012 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. A necessary and sufficient condition that a curve be a straight line is ————

   (a) $k=0$

   (b) $\lambda=0$

   (c) $k=1$

   (d) $k\neq 0$.

2. The ———— at p is the normal in a direction orthogonal to the osculating plane

   (a) normal line (b) binormal line

   (c) osculating line (d) tangent line.

3. The circle which has three point contact with the curve at p is the ————

   (a) osculating circle (b) normal circle

   (c) binormal circle (d) trinormal circle.

4. The radius of spherical curvature is R = ————

   (a) $(e+\sigma e')^{1/2}$ (b) $(e^2+\sigma^2 e'^2)^{1/2}$

   (c) $(e'^2+\sigma^2 e^2)^{1/2}$ (d) $(e^2 e'^2+\sigma^2)^{1/2}$.

5. A point for which ———— is called an ordinary point

   (a) $r_1\times r_2=0$ (b) $r_1\times r_2\neq 0$

   (c) $r_1\times r_2=r$ (d) $r_1\times r_2=1$

6. The parametric curves $u=constant$ are called the ————

   (a) meridians (b) parallels

   (c) normal curves (d) tangent curves.

7. A characteristic property of a geodesic is that at every point its principle normal is ———— to the surface

(a) normal (b) tangent

(c) torsion (d) parallel.

8. A particle is constrained to move on a smooth surface under no force except the normal reaction. Then its path is a ————

(a) straight line

(b) geodesic

(c) helix

(d) circle.

9. If a surface admits two orthogonal families of geodesics, then it is isometric with the ————

(a) surface (b) families

(c) plane (d) space.

10. The geodesic curvature of a geodesic is ————

(a) finite (b) 1

(c) zero (d) $\frac{1}{2}$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the curvature of the curve $\bar{r}=(u,u^2,u^3)$.

Or

(b) Find the equation of the osculating plane of the curve $\bar{r}=\bar{r}(u)$.

12. (a) Show that the necessary and sufficient condition that a curve be a helix is

$$\left[\bar{r}^{IV},\bar{r}^{III},\bar{r}^{II}\right]=-k^5\frac{d}{ds}\left(\frac{\tau}{k}\right)=0.$$

Or

(b) Show that the torsion of an involute of a curve is equal to $\dfrac{e(\sigma e'-\sigma' e)}{(p^2+\sigma^2)(c-s)}$.

13. (a) If w is the angle between two parametric curves, find $\cos w$ and $\sin w$.

Or

(b) For the paraboloid, show that $E=1+4u^2$, $F=-4uv, G=1+4v^2$ and $H=(1+4u^2+4v^2)^{1/2}$.

14. (a) Prove that every helix on cylinder is a geodesic.

Or

(b) State and prove the normal property of geodesic.

15. (a) Obtain the expression of Geodesic curvature Kg in the form Kg$=H(u'\mu - v'\lambda)$.

Or

(b) State and prove Rodrigue's formula.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove the necessary and sufficient condition that a curve be a

(i) Straight line and

(ii) A plane curve.

Or

(b) Derive the curvature and torsion of a curve given as the intersection of two surfaces.

17. (a) Prove that a curve is a helix if and only if $\frac{k}{\tau}$ = constant.

Or

(b) Find the involutes and evolutes of the twisted cubic curve $x=u, y=u^2, z=u^3$.

18. (a) Find the angle between the direction (du, dv) and the parametric are $u=c$.

Or

(b) Find the curvature of a normal section of a helicoid $x=u\cos v, y=u\sin v, z=f(u)+cv$.

19. (a) Prove that the curves bisecting the angle between the parametric curves are given by $E\,du^2 - G\,dv^2 = 0$.

Or

(b) Prove that if θ is the angle at a point (u,v) between the two directions given by $pdu^2 + 2Q\,dudv + R\,dv^2 = 0$ then

$$\tan\theta = \frac{2H\,(Q^2-PR)^{1/2}}{ER-2FQ+GP}.$$

20. (a) State and prove Liouville's formula for Kg.

Or

(b) State and prove Meusnier's theorem.