(6 pages) Reg. No. : ....

Code No. : 7061 **Sub. Code : C 32 M**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2010.

Third Semester

Mathematics

TOPOLOGY — I

(For those who joined in July 2008 and afterwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

1. Define a finite complement topology.

2. Define a homomorphism and topological imbedding.

3. Define a box topology.

4. Define a quotient map.

5. Define a connected space.

6. Define a totally disconnected space.

7. Define the first countability axion.

8. Give an example of a Haurdooff space that is not regular.

9. Define a completely regular space.

10. Prove that a normal space is completely regular.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Set A be a subset of the topological space X. Then prove that $X \in \overline{A}$ if and only if every open set U containing x intersects A.

Or

(b) Show that if A is closed in X and B is closed in Y then A × B is closed in X + Y.

12. (a) Show that the product topology is the Coarest topology on πX_α relative to which each projection function π_β is continuous.

Or

(b) Set $P : X \to Y$ be a quotient map. Let Z be a space and let $g : x \to z$ be a continuous map that is constant on each set $p^{-1}(\{y\})$, for $y \in Y$. Then show that f induces a continuous map $f : Y \to Z$ such that $f\,op = g$.

13. (a) Show that a space X is connected if and only if the only subsets of X that are both open and closed in X are the empty set and X itself.

Or

(b) Prove that every closed subspace of a compact space is compact.

14. (a) Prove that every messizable space is normal.

Or

(b) Set X be a topological space. Set one-point sets in X be closed. Prove that X is regular if and only if given a point x of X and a neighborhood U of x, these is a neighborhood V of x such that $\overline{V} \subseteq U$.

15. (a) Show that the Tietze extension theorem implies the Urysohn lemma.

Or

(b) Prove that product of completely regular spaces is completely regular.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Let X be a topological space. Prove that the following conditions hold :

(i) ϕ and α are closed

(ii) Arbitrary intersection of closed sets are closed.

(iii) Finite unions of closed sets are closed.

Or

(b) Determine whether the following equations hold ; if an equality fails, determine whether one of the inclusion $\supset$ or $\subset$ holds.

(i) $\overline{A \cap B} = \overline{A} \cap \overline{B}$

(ii) $\overline{A - B} = \overline{A} - \overline{B}$

(iii) $(A \cup B)' = A' \cup B'$

(iv) $(A \cap B)' = A' \cap B'$.

17. (a) Set $f : A \to \prod_{2\epsilon s} \times 2$ be given by the equation $f(a) = (f_2(a))_{2\epsilon s}$ where $f_2 : A \to X_2$ for each let. πX_2 have the product topology. Show that the function f is continuous if and only if each f_2 is continuous. Is the above theorem to true is πX_2 is given the box topology? Why?

Or

(b) Set $\{X_2\}$ be a family of spaces : let $A_2 \subset X_2$ for each α.

(i) Show that if A_2 is closed in X_2 then πA_2 is closed in πX_2.

(ii) Show that $\overline{\pi A_2} = \pi \overline{A_2}$.

(iii) Which of (i) and (ii) remain true if you use the box topology instead of the product topology.

18. (a) Prove that the Cartesian product of connected spaces is connected.

Or

(b) Prove the following :

(i) Show that if πX_2 is connected and non empty, then each X_2 is connected.

(ii) Set $\{A_n\}$ be a sequence of connected subsets of X such that $A_n \cap A_{n+1} \neq \phi$ for all n. Show that UA_n is connected.

19. (a) Show that R_l satisfies all the countability axiome but the second.

Or

(b) Show that R_l is normal but the space R_l^2 is not.

20. (a) State and prove the Urysohn meteizion theorem.

Or

(b) State and prove Tietze extension theorem.
