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M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2011.

Second Semester

Mathematics

DIFFERENTIAL EQUATIONS

(For those who joined in July 2008 and afterwards)

Time : Three hours

Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

1. Write the Bernoulli equation.
2. Write the homogeneous equation with constant coefficients.
3. The linear ordinary differential equation $\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$, with v as a constant (real or complex) is called _____.

4. The method of finding the general solution of a second order linear homogeneous equation having a regular singular point at the origin is called _____.
5. Any point that is not an ordinary point of the equation $y'' + P(x)y' + Q(x)y = 0$ is called _____.
6. The general form of the Hermite's equation is _____.
7. The equation which is not homogeneous are called _____.
8. Define the Beta function $\beta(m, n)$ by using the integral formula _____.
9. A solution of a first order ordinary differential equation is also called _____.
10. The special function $H_n^{(1)}(x) = F_n(x) + iy_n(x)$ are called the Bessel functions of the _____.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve : $xy'' - (2x-1)y' + (x-1)y = 0$.

Or

- (b) Find a particular solution of $y'' - 2y' + y = 2x$, first by inspection and then by variation of parameters.

12. (a) Find the power series solution of $y' = 2xy$.

Or

- (b) Show that the series

$$y = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 \cdot 4^2} - \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} + \dots \text{ converges for}$$

all x , and verify that it is a solution of the equation $xy'' + y' + xy = 0$.

13. (a) Determine the nature of the point $x = 0$ for the equation $y'' + (\sin x)y = 0$.

Or

- (b) The equation $x^2y'' - 3xy' + (4x+4)y = 0$ has only one Frobenius series solution. Find it.

14. (a) Find the first three terms of the Legendre series of

$$f(x) = \begin{cases} 0 & \text{if } -1 \leq x < 0 \\ x & \text{if } 0 \leq x \leq 1. \end{cases}$$

Or

- (b) Apply L' Hospital's rule to the limit of the equation

$$y_m(x) = \frac{1}{\pi} \left[\frac{\partial}{\partial p} F_b(x) - (-1)^n \frac{\partial}{\partial p} F - p(x) \right]_{p=m}.$$

15. (a) Replace each of the following differential equation by an equivalent system of first order equations

(i) $y'' - x^2y' - xy = 0$,

(ii) $y''' = y'' - x^2(y')^2$.

Or

- (b) If $w(t)$ is the Wronskian of the two solutions of the homogeneous system, then $w(t)$ is either identically zero or nowhere zero on $[a, b]$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Show that $y = c_1 e^x + c_2 e^{2x}$ is the general solution of $y'' - 3y' + 2y = 0$ on any interval, and find the particular solution for which $y(0) = -1$ and $y'(0) = 1$.

Or

- (b) Show that $y = c_1 e^{2x} + c_2 x e^{2x}$ is the general solution $y'' - 4y' + 4y = 0$ on any interval.

17. (a) The function $w(x, t) = (1 - 2xt + t^2)^{-1/2}$ is the generating function of the Legendre polynomials, in the sense that the expansion $w(x, t) = \sum_{n=0}^{\infty} P_n(x) t^n$ holds sufficiently small values of $|t|$.

Or

- (b) Evaluate $\int_{-1}^1 \log(1-x) P_n(x) dx$.

18. (a) For the differential equation $(3x+1)xy'' - (x+1)y' - 2y = 0$, locate and classify its singular points on the x-axis.

Or

- (b) Determine the nature of the point $x = 0$ for the equation $x^4 y'' + (\sin x)y = 0$.

19. (a) According to the definition, $\mu\left(\frac{1}{2}\right) = \int_0^{\infty} t^{-1}/2 e^{-t} dt$, show that the change of variable $t = s^2$ leads to $\mu\left(\frac{1}{2}\right) = 2 \int_0^{\infty} e^{-s^2} ds$.

Or

- (b) Verify

$$\int \cos^{2n+1} \theta d\theta = \frac{1}{2n+1} \cos^{2n} \theta \sin \theta + \frac{2n}{2n+1} \int \cos^{2n-1} \theta d\theta.$$

20. (a) Find the general solution of the system

$$\begin{cases} \frac{dx}{dt} = x + y \\ \frac{dy}{dt} = 4x - 2y \end{cases} \rightarrow (1)$$

Or

- (b) Show that $f(x, y)y^{1/2}$ does not satisfy a Lipschitz condition on the rectangle $|x| \leq 1$ and $0 \leq y \leq 1$.