(7 pages)

Reg. No. :

Code No.: 5834 Sub. Code: PMAM 14

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2020.

First Semester

Mathematics — Core

ORDINARY DIFFERENTIAL EQUATIONS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — $(10 \times 1 = 10 \text{ marks})$

Answer ALL questions.

Choose the correct answer :

1. Which of the following is the Homogeneous Equation?

(a) y'' = 0 (b) y' = y

(c) y'' = y (d) $y' = e^x$

- 2. If two functions f(x) and g(x) are defined on an interval [a, b] and have the property that one is a constant multiple of the other, then they are said to be on [a, b].
 - (a) Linear
 - (b) Non-linear
 - (c) Linearly dependent
 - (d) Linearly independent
- 3. Which of the following is the Algebraic Equation?

(a)	$\sin x = 0$	(b)	x = 0
(c)	$\cos x = 0$	(d)	$e^x = 0$

- 4. Any point that is not ordinary point of the equation y'' + P(x)y' + Q(x)y = 0 is called
 - (a) Singular point
 - (b) Special function point
 - (c) Ordinary point
 - (d) Point function

5.
$$P_n = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n \quad \text{is called} \qquad -----$$
formula.

- (a) Legendre (b) Rodrigues
- (c) Binomial (d) Bessel
 - Page 2 Code No. : 5834

6.	y = cserie	$a_0 x^m + a_1 x^{m+1} + \dots$ es.	is	called ———
	(a)	Frobenius	(b)	Rodrigues
	(c)	Binomial	(d)	Bessel
7.	$\Gamma(n)$)=		
	(a)	0	(b)	1
	(c)	n!	(d)	(n-1)!
8.	$\Gamma\left(\frac{1}{2}\right)$) =		
	(a)	π	(b)	$\sqrt{\pi}$
	(c)	0	(d)	∞
9.	If $W(t)$ is the Wronskian of the two solutions the homogeneous system then $W(t)$ is ———— on $[a, b]$.			
	(a)	Identically zero	(b)	Never zero
	(c)	Either (a) or (b)	(d)	Zero
10.	The syste	system $\frac{dx}{dt} = a(t)x$ em is called	+f(t)), $f(t) \equiv 0$ then this
	(a)	Homogeneous	(b)	Non-Homogeneous
	(c)	Non-linear	(d)	Wronskian

Page 3 Code No. : 5834

PART B — $(5 \times 5 = 25 \text{ marks})$

Answer ALL questions, choosing either (a) or (b).

- 11. (a) If $y_1(x)$ and $y_2(x)$ are any two solutions of
 - y'' + P(x)y' + Q(x)y = 0, then prove that $C_1 y_1(x) + C_2 y_2(x)$ is also a solution for any constants C_1 and C_2 .

Or

(b) Find the general solution for

$$x^2y'' + xy' - y = 0.$$

12. (a) Prove that the equation x'' - 2tx' + 2x = 0 has an ordinary point at t = 0.

Or

- (b) Find the general solution for y'' + y = 0.
- 13. (a) Discuss the nature of Singularity of $f(z) = \frac{e^z}{z}$.

Or

(b) Discuss the nature of Singularity of $f(z) = \frac{\sin(z-a)}{(z-a)^4}$.

Page 4	Code No. : 5834
	[P.T.O.]

14. (a) Prove that $J_{-n}(x) = (-1)^n J_n(x)$.

Or

- (b) Find the general solution of the equation $9x^{2}y'' + 9xy' + \left(9x^{2} - \frac{1}{4}\right)y = 0.$
- 15. (a) Find the Wronskian value of $W(x, x^2, x^3)$.

Or

(b) Find the general solution of the system x' + Ax.

PART C — $(5 \times 8 = 40 \text{ marks})$

Answer ALL questions, choosing either (a) or (b).

16. (a) Solve $x^2y'' + 2xy' - 2y = 0$.

Or

(b) If $y_1(x)$ and $y_2(x)$ are two solutions of y'' + P(x)y' + Q(x)y = 0 on [a, b], then prove that they are linearly dependent on this interval if and only if their Wronskian $W(y_1, y_2) = y_1 y'_2 - y_2 y'_1$ is identically zero.

Page 5 Code No. : 5834

17. (a) Find the power series solution for the equation x' + x = t, x(0) = 0.

Or

(b) Find the power series solution for the equation (1+x)y' = Py, y(0) = 1.

18. (a) Consider the equation

 $t(t-1)^2(t+3)x''+t^2x'-(t^2+t-1)x=0$. Check whether the point t = 0 is the regular Singular point or not.

Or

- (b) If P_n is the Legendre polynomial, then prove that $\int_{-1}^{1} P_n^2(t) dt = \frac{2}{2n+1}$.
- 19. (a) Consider the differential equation $4x^2y'' + 4xy' + \left(x - \frac{1}{36}\right)y = 0$. Set $Z = \sqrt{x}$ and reduce the differential equation to a Bessel equation in $Z, \frac{dy}{dz}$ and $\frac{d^2y}{dz^2}$.

Or

(b) Prove that $P'_n(1) = \frac{1}{2}n(n+1)$. Page 6 **Code No. : 5834**

- 20. (a) Find the general solution of $\frac{dx}{dt} = x + y$; $\frac{dy}{dt} = 4x - 2y$. Or
 - (b) Find the general solution of $\frac{dx}{dt} = 3x 4y$; $\frac{dy}{dt} = x - y$.

Page 7 Code No. : 5834