(6 pages) **Reg. No.:.............................**

Code No. : 5854 **Sub. Code : PMAM 43**

M.Sc. (CBCS) DEGREE EXAMINATION,
APRIL 2020.

Fourth Semester

Mathematics — Core

ADVANCED ALGEBRA – II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer

1. If Q is the field of rational numbers, then $\left[Q\left(\sqrt{2}\right):Q\right]+\left[Q\left(\sqrt{3}\right):Q\right]$ is

   (a) 2 (b) 6

   (c) 4 (d) 5

2. If L is a finite extension of F and K is a subfield of L which contains F, then

(a) $[L:K]/[L:F]$ (b) $[K:F]/[L:F]$

(c) $[L:F]/[L:K]$ (d) $[L:F]/[K:F]$

3. 2 s a root of $(x-2)^4\ (x+3)^5\left(x^2-3x+2\right)$ of multiplicity

(a) 4 (b) 3

(c) 5 (d) 11

4. The characteristic of the field of rotational numbers Q is

(a) A prime number P (b) 0

(c) 1 (d) A composite number

5. If G is a group of automorphisons of K, then the fixed field of G is

(a) $\{a \in k / \sigma\ (a) = a \forall \sigma \in G\}$

(b) $\{a \in k / \sigma\ (a) = o \forall \sigma \in G\}$

(c) $\{a \in a / \sigma\ (a) = a \forall \sigma \in K\}$

(d) $\{a \in a / \sigma\ (a) = 0 \forall \sigma \in K\}$

6. If $K=\mathbb{C}$ and $F=\mathbb{R}$, then the fixed field of $G(K, F)$ is

(a) K

(b) F

(c) a filed between K and F

(d) ϕ

7. Which one of the following is a filed?

(a) J_6 (b) J_{18}

(c) J_{28} (d) J_{19}

8. The cyclotornic polynomial $\Phi_4(x)$ is

(a) x^2+1 (b) $x-1$

(c) $x+1$ (d) x^2+x+1

9. Let H be the Hurwitz ring of integral quaternions. If $a \in H$ then $a^{-1} \in H$ if and only if.

(a) $N(a)=0$ (b) $N(a)=1$

(c) $N(a)=\pm 1$ (d) $N(a) \neq 0$

10. Let C be the field of complex numbers and suppose that the divisions ring D is algebraic are C. Then

(a) $D \neq C$ (b) $D = C$

(c) $C \underset{+}{\subset} D$ (d) $D \underset{+}{\subset} C$

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If L is an algebraic of K and if K if K an algebraic extension of F, prove that L is an algebraic extension of F.

Or

(b) If a, $b \in K$ are algebraic over F of degree on and n, respectively, and if m and n are relatively prime, prove that $f(a,b)$ is of degree mn over F.

12. (a) Prove that a polynomial of degree n over a field can have at most n roots in any extension field.

Or

(b) If $f(x)$ and $f'(x)$ have a non trivial common factor, prove that $f(x) \in F[x]$ has a multiple root.

13. (a) Prove that $G(K, F)$ is a subgroup of the group of all automorphism of K.

Or

(b) If K is finite extension of F, prove that $O(G(K,F)) \leq [K:F]$.

14. (a) For any prime number P and any integer m, prove that there is a field having p^m elements.

Or

(b) Let G be a finite abelian group enjoying the property that the relation $x^n = e$ is satisfied by at most n elements of G, for every integer n, prove that G is a cyclic group.

15. (a) Suppose that the division ring D is algebraic over the field of complex number D, prove that $D = C$.

Or

(b) Let Q be the divisions ring of real quaternions for $x \in Q$, define $N(x)$ and prove that $N(xy) = N(x)\,N(y)$for all $x, y \in Q$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b)

16. (a) With usual notations, prove that $[L:F] = [L:K][K:F]$.

Or

(b) Prove that the element of $a \in k$ is algebraic over F if and only if $F(a)$ is a finite extension of F.

17. (a) If $P(x)$ is irreducible in $F[x]$ and if v is a root of $p(x)$, prove that $F(v)$ is isomorphic to $F'(w)$ where w is a root of $p'(t)$.

Or

(b) If F is of characteristic O and if a,b and algebraic over F, prove that there exist an element $C \in F(a,b)$ such that $F(a,b) = F(c)$.

18. (a) Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F.

Or

(b) State and prove the fundamental theorem of Galois theory.

19. (a) If F is a finite field and $\alpha \neq o$, $\beta \neq 0$ and two elements, of F, prove that we can find elements a and b in F such that $1 + \alpha a^2 + \beta b^2 = 0$.

Or

(b) Prove that a finite divisions ring is necessarily a commutative field.

20. (a) State and prove Frobenius theorem.

Or

(b) Prove that every positive integer can be expressed as the sum of squares of four integers.
