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M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2017.

Second Semester

Mathematics/Mathematics with Computer
Application

ANALYSIS – II

(For those who joined in July 2016 onwards)

Time : Three hours

Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Let α be continuous on $x_0 \in [a, b]$. Let $f(x_0) = 1$, $f(x) = 0$ for $x \neq x_0$. Then
- (a) f is Riemann Integrable for all values in $[a, b]$
 - (b) f is not Riemann integrable over $[a, b]$
 - (c) $\int f d\alpha = x_0$
 - (d) $\int f d\alpha = 1$

2. Let $f(x) = \begin{cases} 0, & x \text{ is irrational} \\ 1, & x \text{ is rational} \end{cases}$ then
- (a) f is Riemann integrable on $[a, b]$; $a < b$
 - (b) f is not Riemann integrable on $[a, b]$; $a < b$
 - (c) f is continuous on $[a, b]$
 - (d) f is constant on $[a, b]$

3. Let $s_{m,n} = \frac{m}{m+n}$, then for every fixed n , $\lim_{n \rightarrow \infty} s_{m,n} =$
- (a) 1
 - (b) 0
 - (c) s_n
 - (d) $\frac{1}{n}$

4. The value of $\int_0^1 x(1-x^2)^n dx =$
- (a) 1
 - (b) $\frac{1}{2}$
 - (c) $\frac{1}{2n+2}$
 - (d) $\frac{1}{2n-2}$

5. An example of an algebra which does not separate points is
- (a) set of all even polynomials on $[-1, 1]$
 - (b) set of all odd polynomials on $[-1, 1]$
 - (c) all polynomials in one variable
 - (d) set of constant polynomials in $[0, 1]$

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6. A family of complex functions defined on a set E is an algebra if

- (a) it is closed under addition
- (b) it is closed under multiplication
- (c) it is closed under scalar multiplication
- (d) (a), (b) and (c) hold

7. The value of $\lim_{x \rightarrow 0} \frac{\log(1+x)}{x}$ is

- (a) 1
- (b) 0
- (c) ∞
- (d) $\frac{1}{x}$

8. The value of $\lim_{x \rightarrow 0} (1+x)^{1/x}$ is

- (a) 1
- (b) 0
- (c) $1+x$
- (d) e

9. The sum $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is

- (a) π
- (b) $\frac{\pi^2}{2}$
- (c) $\frac{\pi^2}{4}$
- (d) $\frac{\pi^2}{6}$

10. The sum of the series $\frac{1}{n^4}$ is

- (a) $\frac{\pi^4}{9}$
- (b) $\frac{\pi^4}{10}$
- (c) $\frac{\pi^4}{90}$
- (d) $\frac{\pi^4}{180}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let f be a bounded function on $[a, b]$ with finitely many points of discontinuity. Then show that $f \in R(\alpha)$ where α is continuous at every point at which f is discontinuous.

Or

(b) Let f be continuous function on $[a, b]$. Prove that $f \in R(\alpha)$ on $[a, b]$.

12. (a) State and prove Weierstrass test for uniform convergence of functions.

Or

(b) Let $f_n(x) = \frac{\sin nx}{\sqrt{n}}$; $x = \text{real}$; $n = 1, 2, 3, \dots$

Show that $\{f_n\}$ does not converge to f' .



13. (a) Let K be a compact metric space, if $f_n \in \mathcal{C}(K)$ and $\{f_n\}$ converges uniformly on K then show that $\{f_n\}$ is equicontinuous on K .

Or

- (b) Let $\mathcal{B}$ be the uniform closure of an algebra $\mathcal{A}$ of bounded functions. Then $\mathcal{B}$ is a uniformly closed algebra.

14. (a) Show that $\lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n = e^x$.

Or

- (b) If $0 < x < \pi/2$, show that $\frac{2}{\pi} < \frac{\sin x}{x} < 1$.

15. (a) Let f be a continuous function & $\epsilon > 0$. Show that there exists a trigonometric polynomial P such that $|P(x) - f(x)| < \epsilon$ for all real x .

Or

- (b) Define Gamma function and show that
 (i) $\Gamma(x+1) = \Gamma(x)x$; $0 < x < \infty$
 (ii) $\Gamma(n+1) = n!$; $n = 1, 2, 3, \dots$
 (iii) $\log \Gamma$ is convex on $(0, \infty)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove the fundamental theorem of calculus for Riemann integrable functions.

Or

- (b) State and prove the integration by parts rule.

17. (a) Let X be a metric space, $\mathcal{C}(X)$ denote the set of all complex-valued, continuous, bounded functions on X . Prove that $\mathcal{C}(X)$ is a complete metric space.

Or

- (b) Let $\{f_n\}$ be a sequence of continuous functions on a set E and $f_n \rightarrow f$ uniformly on E . Prove that f is continuous on E .

18. (a) Let K be compact, $f_n \in \mathcal{C}(K)$ and $\{f_n\}$ is pointwise bounded and equicontinuous on K . Prove that $\{f_n\}$ is uniformly bounded and contains a uniformly convergent subsequence.

Or

- (b) Let f be a continuous complex function on $[a, b]$. Prove that there exists a sequence of polynomials P_n such that $\lim_{n \rightarrow \infty} P_n(x) = f(x)$ uniformly on $[a, b]$.



19. (a) Show that the function e^x is
- (i) Strictly increasing function of x
 - (ii) Continuous and differentiable for all x
 - (iii) $e^x \rightarrow \infty$ as $x \rightarrow \infty$, $e^x \rightarrow 0$ as $x \rightarrow -\infty$
 - (iv) $(e^x)' = e^x$.

Or

- (b) Give the series expansion of e^x and prove that

- (i) $e^{x+y} = e^x \cdot e^y$
- (ii) $\lim_{x \rightarrow +\infty} x^n e^{-x} = 0 \forall n$
- (iii) $(e^x)' = e^x$.

20. (a) Define Gamma function and prove any three properties.

Or

- (b) State and prove Stirling's formula.
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