(7 pages) **Reg. No. :** ....

Code No. : 41162 E **Sub. Code : JMMA 64/ JMMC 64**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Sixth Semester

Mathematics/Mathematics with CA – Main

GRAPH THEORY

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The number of edges in K_6 is ————.

   (a) 36 (b) 30

   (c) 15 (d) 12

2. Which of the following is not true?

   (a) Every walk is a path

   (b) Every path is a trail

   (c) Every trail is a walk

   (d) Every path is a walk

3. If we remove the cut vertices from a graph G, then the number of components

   (a) decreases

   (b) increases

   (c) no change

   (d) nothing can be said

4. The number of edges in a tree with 20 vertices is

   (a) 20 (b) 21

   (c) $\frac{20 \times 19}{2}$ (d) 19

5. The smallest nm planar complete graph is

(a) K_3 (b) K_4

(c) K_5 (d) K_6

6. If G is a (p, q) plannan graph with f faces, then $p - q + f =$ ————.

(a) 1 (b) 2

(c) 3 (d) 4

7. The rank of cut set matrix with p vertices is ————.

(a) $\leq p-1$ (b) $= p-1$

(c) $\geq p-1$ (d) $= p+1$

8. The $(i,i)^{th}$ entry of the adjauncy matrix of a graph ————

(a) 1 (b) 0

(c) 2 (d) –1

9. Chromatic number of $K_{m,n}$ is ————.

(a) p (b) $p-1$

(c) 1 (d) 2

10. Chromatic polynomial $f(K_1, \lambda) =$ ————.

(a) λ (b) λ^2

(c) $\lambda^2 + 1$ (d) $\lambda^2 + \lambda$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that the sum of the degrees of the points of any graph is twice the number of lines.

Or

(b) A graph G is connected iff for any partition of V into subsets V_1 and V_2 there is a line of G joining a point of V_1 to a point of V_2.

12. (a) Prove that every tree has either one or two centers.

Or

(b) Prove that every connected graph has a spanning tree.

13. (a) Prove that K_5 is non-planon.

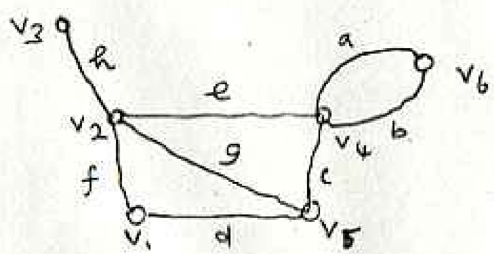
Or

(b) If G is a (p, q) planon graph in which every face is an n cycle, prove that $q = \frac{n(p-2)}{n-2}$.

14. (a) Let G_1 be a (p_1, q_2) graph and G_2 be (p_2, q_2) graph. Then prove that $G_1 \times G_2$ is a $(p_1 p_2, q_1 p_2 + q_2 p_1)$ graph.

Or

(b) Find the incidence matrix for the given graph.



15. (a) If G is a tree with $n \geq 2$ points, prove that the chromatic polynomial $f(G, \lambda) = \lambda(\lambda_{-1})^{n-1}$.

Or

(b) Prove that every k-chromatic graph has atleast k vertices of degree atleast $k-1$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove Dirac's theorem.

Or

(b) Prove that a simple graph with p vertices and k components can have atmost $\frac{(p-k)(p-k+1)}{2}$ edges.

17. (a) Prove that a connected graph has p vertices and $p-1$ edges iff it is a tree.

Or

(b) For any graph G, prove that vertex connectivity $\leq$ line connectivity $\leq \delta$.

18. (a) Prove that a connected planar graph with p vertices and q edges has $q - p + 2$ regions.

Or

(b) Write down the relationship between the planar graph and its dual.

19. (a) Write remarks on adjacency matrix.

Or

(b) Prove that the rank of cut set matrix $C(G)$ = the rank of incidence matrix $A(G)$ = rank of graph G.

20. (a) State and prove Five colour theorem.

Or

(b) If d_{max} is the maximum degree of the vertices in a graph G, then prove that chromatic number of $G \leq 1 + d_{max}$.
