(7 pages) **Reg. No. :** .....

Code No. : 41383 E **Sub. Code : SMMA 21**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Second Semester

Mathematics — Main

ANALYTICAL GEOMETRY OF THREE DIMENSIONS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer.

1. The equation of the tangent whose vertical angle ϕ is

(a) $\frac{l}{r} = \rho\cos\theta - \cos(\theta - \phi)$

(b) $\frac{l}{r} = \rho\cos\theta + \cos(\theta - \phi)$

(c) $\frac{l}{r} = \rho\cos\theta - \sin(\theta - \phi)$

(d) $\frac{l}{r} = \rho\cos\theta + \sin(\theta - \phi)$

2. The asymptotes of the conic $\frac{l}{r} = 1 + e\cos\theta$ is

(a) $\frac{e^2-1}{e}\left\{\cos\theta \pm \frac{\sin\theta}{\sqrt{e^2-1}}\right\}$

(b) $\frac{e^2-1}{e}\left\{\sin\theta \pm \frac{\sin\theta}{\sqrt{e^2-1}}\right\}$

(c) $\frac{e^2+1}{e}\left\{\cos\theta \pm \frac{\sin\theta}{\sqrt{e^2-1}}\right\}$

(d) None

3. $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n} = r$ is the equation of the _____.

(a) Circle (b) Straight line

(c) Ellipse (d) Hyperbola

4. $x^2 + y^2 + z^2 - 2x + 6y + 4z - 35 = 0$ is then the centre is, _____.

(a) $(-1, 3, 2)$ (b) $(1, -3, -2)$

(c) $(-2, 6, 4)$ (d) $(2, -6, -4)$

5. When a sphere is a cut by a plane through its centre _____ is obtained.

(a) Circle (b) Ellipse

(c) Parabola (d) Hyperbola

6. A cylinder is a surface generated by ————————.

(a) Straight line (b) Sphere

(c) Circle (d) None

7. The fixed distance of the right circular cylinder is

(a) Semi latus rectum (b) Radius

(c) Axis (d) None

8. The constant angle of right circular cone is

(a) acute angle (b) semi vertical

(c) right angle (d) none

9. Equation of hyperbolic of one sheet is

(a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ (b) $\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

(c) $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ (d) None

10. The intersection of $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ ————————.

(a) a parabola (b) an ellipse

(c) a hyperbola (d) a circle

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Show that the points $(5, 3, -2)$, $(3, 2, 1)$ and $(-1, 0, 7)$ are collinear.

Or

(b) Find the angle between two diagonals of a cube.

12. (a) Derive the equation of the plane passing through the points (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3).

Or

(b) Find the distance between the parallel planes $2x-2y-z=3$ and $4x-4y+2z+5=0$.

13. (a) Find the equation of the image of the line $\frac{x-1}{2}=\frac{y+2}{-5}=\frac{z-3}{2}$ in the plane $2x-3y+3+2z=0$.

Or

(b) Find the shortest distance between the lines $\frac{x-3}{-1}=\frac{y-4}{2}=\frac{z+2}{1}$; $\frac{x-1}{1}=\frac{y+7}{3}=\frac{z+2}{2}$.

14. (a) A sphere of constant radius k passes through the origin and meets the axes in A, B, C. Prove that the centroid of the triangle ABC lies on the sphere $9(x^2+y^2+z^2)=4k^2$.

Or

(b) Show that the plane $2x-y-2z=16$ touches the sphere $x^2+y^2+z^2-4x+2y-2z-3=0$ and find the point of contact.

15. (a) Find the equation of the cone of the second degree with passes through the axes.

Or

(b) Find the equation of a right circular cylinder of radius 3 with axis $\frac{x+2}{3}=\frac{y-4}{6}=\frac{z-1}{2}$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Show that the four points $(4,-1,2)$, $(0,-2,3)$, $(1,-5,-1)$ and $(2,0,1)$ lie on a sphere whose centre is $(2,-3,1)$. Find the radius of the sphere.

Or

(b) Show that the straight lines whose direction cosines are given by $al+bm+cn=0$, $fmn+gnl+hlm=0$ are perpendicular if $\frac{f}{a}+\frac{g}{b}+\frac{h}{c}=0$ and parallel if $\sqrt{af}+\sqrt{hg}+\sqrt{ch}=0$.

17. (a) Show that the following points are coplanar and find the equation of the plane on which they lie

(i) $(0,-1,-1)$, $(-4,4,4)$, $(4,5,1)$ and $(3,9,4)$

(ii) $(0,2,-4)$, $(-1,1,-2)$, $(-2,3,3)$ and $(-3,-2,1)$.

Or

(b) Show that the equation

$x^2+y^2+4z^2+4yz+4zx+2xy+7(x+y+2z)+12=0$

represents a pair of parallel planes and find the distance between them.

18. (a) The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the axes in A, B, C. Find the coordinates of the orthocentre of the triangle ABC.

Or

(b) Prove that the lines $\frac{x+1}{-3}=\frac{y+10}{8}=\frac{z-1}{2}$; $\frac{x+3}{-4}=\frac{y+1}{7}=\frac{z-4}{1}$ are coplanar. Also their points intersection and the plane through them.

19. (a) A plane passes through a fixed point (a, b, c) and cuts the axes A, B, C show that the locus of the centre of the sphere $OABC$ is $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$.

Or

(b) The plane ABC, whose equation is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the axes in A, B, C. Find the equation to the circumcircle of the triangle ABC and obtain the coordinates of its centre and radius.

20. (a) Find the condition for the equation

$$F(x, y, z)=ax^2+by^2+cz^2+2fyz+2gzx+2hxy+2ux+2vy+2wz+d=0$$

to represent a cone.

Or

(b) Find the equation of the right circular cylinder described on the circle through the points $(a, 0, 0)$, $(0, a, 0)$, $(0, 0, a)$ as a guiding curve.
