(7 Pages)

**Reg. No. :** .....

**Code No. : 20547 E** **Sub. Code : JACA 21/ SACA 21/AACA 21**

B.C.A. (CBCS) DEGREE EXAMINATION, APRIL 2021.

Second Semester

Computer Application — Allied

MATHEMATICAL FOUNDATION FOR COMPUTER SCIENCE

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. Let $A = \{1, 2, 3\}$ and $B = \{a, b\}$. Then $A \times B$ consists of ———— elements.

(a) 3 (b) 4

(c) 5 (d) 6

2. If $A = \varphi$ and $B = \{2, 3, 4\}$, then $A \cap B =$ ————

(a) $\varphi$ (b) $\{2\}$

(c) $\{3\}$ (d) $\{4\}$

3. Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and $f = \{(1, 4), (2, 5), (3, 6)\}$. Then $f$ is ———— function.

(a) one-to-one (b) onto

(c) constant (d) identity

4. $f : R \to R$ given by $f(x) = 5$ is called a ——— function.

(a) identity (b) bijective

(c) constant (d) onto

5. The contrapositive statement of $P \to Q$ is ———

(a) $Q \to P$ (b) $\neg P \to \neg Q$

(c) $\neg Q \to \neg P$ (d) $P \to \neg Q$

6. $P \vee \neg P \Leftrightarrow$ ————

(a) $P$ (b) $T$

(c) $F$ (d) $\neg P$

7. A graph that has neither self-loops nor parallel edges is called ———— graph.

(a) multi (b) trivial

(c) simple (d) regular

8. A vertex of degree one is called ———— vertex.

(a) isolated (b) pendant

(c) path (d) null

9. A tree is connected ———— graph.

(a) Acyclic (b) Cyclic

(c) Euler (d) Hamiltonian

10. The number of vertices in a binary tree is always ————

(a) even (b) odd

(c) center (d) path

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) In an examination 75% of the students passed in Physics and 85% passed in Chemistry 70% passed in both. What percentage of the students failed in both?

Or

(b) Show that the relation $\rho$ in the set $Z$ of integers given by $\rho = \{(a, b) : 2 \text{ divides } a - b\}$ is an equivalence relation.

12. (a) Explain the different types of functions with example.

Or

(b) Let $f : X \to Y$ be a bijection. Prove that $f^{-1} : Y \to X$ is also a bijection and $f \circ f^{-1} = I_y$ and $f \circ f^{-1} = I_x$.

13. (a) Show that $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$.

Or

(b) Construct the truth table to show that $(P \to Q) \leftrightarrow (\neg Q \to \neg P)$ is a tautology.

14. (a) Let $G$ be a graph with $n$ vertices and $e$ edges. The degree of vertices in the graph is either $l$ or $l + 1$. If $p$ ($p > 0$) vertices of the graph have degree 1, show that $p = n\ (l + 1) - 2\ e$.

Or

(b) Define :

(i) regular graph

(ii) complete graph

(iii) bipartite graph with example.

15. (a) Prove that every walk in a graph $G$ contains a path.

Or

(b) Prove that the number of pendant vertices in a binary tree is $(n + 1)/2$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) For any three sets $A$, $B$, $C$ prove the following :

(i) $(A - B) - C = A - (B \cup C)$

(ii) $A - (B - C) = (A - B) \cup (A \cap C)$

Or

(b) Show that if $A$ and $B$ are any two sets, then $A - B$, $B - A$ and $A \cap B$ are pairwise disjoint.

17. (a) Which of the following functions are bijection? If $f$ is a bijection find $f^{-1}$

(i) $f : R \to R$ defined by $f(x) = x + 5$

(ii) $f : R \to R$ defined by $f(x) = 3x$

(iii) $f : R^* \to R^*$ defined by $f(x) = \frac{1}{x}$

Or

(b) Construct a function such that

(i) $f$ is 1-1 but not onto

(ii) $f$ is into but not 1-1

(iii) $f$ is both 1-1 and onto

(iv) $f$ is neither 1-1 nor onto

18. (a) Check whether the following formulae is a tautology or a contradiction

$\neg(P \vee (Q \wedge R)) \leftrightarrow (P \vee Q) \wedge (P \vee R)$

Or

(b) Obtain the principal disjunctive normal form of

$(P \rightarrow (Q \vee R)) \wedge (\neg Q) \wedge (\neg R) \wedge Q$

19. (a) Explain the following with example :

(i) weighted graph

(ii) directed graph

(iii) undirected graph

(iv) mixed graph

Or

(b) Prove the following :

(i) the number of vertices of odd degree in a graph $G$ is always even.

(ii) if a simple graph $G$ has at least two vertices, then there at least two vertices that have the same degree.

20. (a) Prove that a connected graph $G$ is Eulerian iff every vertex has an even degree.

Or

(b) Prove that a graph $G$ is connected iff for any partition of $V$ into disjoint subsets $V_1$ and $V_2$ there is an edge of $G$ joining a vertex of $V_1$ to a vertex of $V_2$.

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