(6 pages) **Reg. No. :** ....

Code No. : 6840 **Sub. Code : PMAM 25**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2021.

Second Semester

Mathematics — Core

GRAPH THEORY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. If G is a forest, then the number of edges is

   (a) $\varepsilon = \gamma - \omega$ (b) $\varepsilon = \gamma - 2$

   (c) $\varepsilon = \gamma$ (d) $\varepsilon = \frac{\gamma}{2}$

2. If any two vertices of G are connected by atleast two internally disjoint paths, then G is

   (a) 2-connected (b) 3-connected

   (c) 4-connected (d) 5-connected

3. A connected graph has an Euler trail if it has atmost ___________ vertices of odd degree.

(a) 3 (b) 2

(c) 1 (d) 4

4. The number of Eulerian graphs with γ even and ε odd is

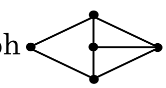
(a) 1 (b) 2

(c) 0 (d) $\gamma\varepsilon$

5. Which one of the following is not 1-factorable?

(a) Petercen Graph (b) $K_{5.5}$

(c) K_{10} (d) None of these

6. The edge-chromatic number of the graph  is

(a) 1 (b) 2

(c) 3 (d 4

7. G is a graph on 12 vertices. The covering number of G is 4. Then the independence number of G is

(a) 16 (b) 48

(c) 8 (d) 3

8. $\chi(K_4, 4) =$

(a) 1 (b) 2

(c) 4 (d) 0

9. If G is a loopless graph with $\Delta = 3$, then χ'

(a) $= 3$ (b) $= 2$

(c) < 4 (d) ≤ 4

10. If G is 4-chromatic, the G contains a subdivision of

(a) K_1 (b) K_2

(c) K_3 (d) K_4

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, by choosing either (a) or (b).

11. (a) Prove that every tree has either one center or two adjacent centers.

Or

(b) Prove that a connected graph G is a tree if and only if every edge of G is a cut edge.

12. (a) Prove that a connected graph has an Euler trial if and only if it has atmost two vertices of odd degree.

Or

(b) Explain the traveling salesman problem.

13. (a) Prove that every 3-regular graph without cut edges has a perfect matching.

Or

(b) Let M and N be disjoint matchings of G with $|M| > |N|$. Prove that there are disjoint matchings M^1 and N^1 of G s.t. $|M^1| = |M| - 1$, $|N^1| = |N| + 1$ and $M^1 \cup N^1 = M \cup N$.

14. (a) For any two positive integers k and l, prove that $r(k, l \geq 2^{m/2})$ where $m = \min\{k, l\}$.

Or

(b) State and prove Turan's Theorem.

15. (a) If G is a K-Critical, prove that $\delta \geq K - 1$.

Or

(b) Prove that every critical graph is a block.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, by choosing either (a) or (b).

16. (a) Prove that $\tau(K_n) = n^{n-2}$.

Or

(b) Prove that the spanning tree obtained by Kruskal's algorithm is an optimal tree.

17. (a) Let G be a simple graph with degree sequence $(d_1, d_2, ..., d_\gamma)$ where $d_1 \leq d_2 \leq .... \in d_\gamma$ and $\gamma \geq 3$. Suppose that there is no value of $m < \frac{\gamma}{2}$ for which $d_m \leq m$ and $d_{8-m} < \gamma - m$. Prove that G is Hamiltonian.

Or

(b) If G is Eulerian, prove that any trial in G constructed by Fleury's algorithm is an Euler Tour of G.

18. (a) State and prove Hall's theorem.

Or

(b) Prove that in a bipartite graph, the number of edges in a maximum matching is equal to the number of vertices in minimum covering.

19. (a) In any graph G with $\delta(G) > 0$, prove that $\alpha^1 + \beta^1 = \gamma$.

Or

(b) For any two integers $k \geq 2$, $l \geq 2$, prove that $r(k, l) \leq r(k, l-1) + r(l-1, 2)$. If $r(k, l-1)$ and $r(k-1, l)$ are both even, prove that strict inequality holds.

20. (a) State and prove Brook's theorem.

Or

(b) If G is a tree, prove that $\pi_k(G) = k(k-1)^{\gamma-1}$ and hence find the chromatic polynomial of