(8 Pages) **Reg. No. :** ....

Code No. : 30369 E **Sub. Code : SEMA 6 E**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2022

Sixth Semester

Mathematics

Major Elective — CODING THEORY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. We will call one channel more reliable them another if its reliability is ————
   (a) lower (b) higher
   (c) equal (d) not equal

2. Let C be the code of all words of length 3. Then which one of the following code-word is most likely sent if 001 received is ————
   (a) 010 (b) 100
   (c) 001 (d) 011

3. Two vectors v and w in K^n are orthogonal if ————
   (a) $v.w = 1$ (b) $v.w \neq 1$
   (c) $v.w = 0$ (d) $v + w = 0$

4. The dimension of the code $C = \{000, 010, 011, 111, 001, 101, 100, 110\}$ is ————
   (a) 1 (b) 3
   (c) 4 (d) 2

5. The distance of a linear code C with parity – check x matrix, $H = \begin{bmatrix} 110 \\ 011 \\ 100 \\ 010 \\ 001 \end{bmatrix}$ is ————
   (a) 2 (b) 3
   (c) 4 (d) 1

6. The number of cosets of a linear code C of length 4 and has dimension 2 is ————
   (a) 1 (b) 2
   (c) 3 (d) 4

7. The dual of an $(n,K,n-K+1)$ MDS code is an ________ MDS code.

(a) $(n,K,n-K)$

(b) $(n,n-K+1,K+1)$

(c) $(n-K,K,K+1)$

(d) $(n,n-K,K+1)$

8. If the distance d of a linear code C is odd, then the distance of its extendened code C^* is ________

(a) $d+1$ (b) $d-1$

(c) d (d) $2d+1$

9. If $f(x)=1+x+x^3+x^4$ and $g(x)=x+x^2+x^3$ be the polynomials in $K[x]$, then $f(x)+g(x)$ is ________

(a) $1+x^2+x^3$ (b) $1+x^2+x^4$

(c) $x+x^2+x^3$ (d) $x+x^2+x^4$

10. The cycle shift of the word 101101 is ________

(a) 101101 (b) 110110

(c) 110011 (d) 011011

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the maximum number of codewords of length 4 in a cods in which any single error can be detected.

Or

(b) If $c=\{01000, 01001, 00011, 11001\}$ and a word $w=10110$ is received, which codeword is most likely to have beensent?

12. (a) Find the dual code $C^{\perp}$ of a linear code $C=<S>$, where $S=\{0100, 0101\}$.

Or

(b) Test the set $S=\{110, 011, 101, 111\}$ for linear independence.

13. (a) Find the distance of a linear code C with parity – check matrix.

$$H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Or

(b) For a code $C = \{0000, 1011, 0101, 1110\}$ and for each coset, calculate the syndrome using the coset leader.

14. (a) Does there exist a linear code of length $n = 9$, dimension $k = 2$ and distance $d = 5$?

Or

(b) Can there exist perfect codes for the values $n = 7$ and $d = 3$.

15. (a) Find the quotient and remainder when $h(x) = 1 + x + x^2 + x^4$ is divided into $f(x) = x + x^2 + x^6 + x^7 + x^8$.

Or

(b) Find a basis for the smallest linear cyclic code of length n, containing $v = 0101$ and $n = 4$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Explain how to convert with $0 < p \le \frac{1}{2}$ into a channel with $\frac{1}{2} \le p < 1$.

Or

(b) Which of the code words 110110, 110101, 000111, 100111, 101000 is most likely to have been sent if $w = 011001$ is received.

17. (a) Find two different bases for the linear code $C = < S >$, where $S\{11101, 10110, 01011, 11010\}$.

Or

(b) Use the algorithm for finding the basis for dual code $C^{\perp}$, find the basis for $C^{\perp}$ for each of the codes $C = < S >$, where

(i) $S = \{010, 011, 111\}$

(ii) $S = \{1010, 0101, 1111\}$

(iii) $S = \{0101, 1010, 1100\}$.

18. (a) List the cosets each of the linear code C with generator matrix.

(i) $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$

(ii) $G = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$.

Or

(b) Let $C = \{0000, 1011, 0101, 1110\}$ be the code of length 4. Use the procedure for CMLD just outlined to decode each of the following received words.

(i) 000011

(ii) 001001

(iii) 001101.

19. (a) For a (n,k,d) linear code C prove that the following are equivalent :

(i) $d = n - k + 1$

(ii) every $n-k$ rows of the parity – check matrix are linearly independent

(iii) every k – columns of the generator matrix are linearly independent and

(iv) C is MDS.

Or

(b) The code is C_{24}. Decode, if possible, each of the following received words w.

(i) 001001001101, 101000101000

(ii) 111000000000, 011011011011.

20. (a) Let $g(x) = 1 + x + x^3$ be the generator polynomial of a linear cyclic code of length 7. Encode the following message polynomials : $1 + x^2$, $1 + x + x^4 + x^6$.

Or

(b) Find all idempotents polynomial and find the corresponding generator polynomial for the cyclic codes of length 9.
