(7 pages) **Reg. No. :** ....

Code No. : 8368 **Sub. Code : HMAM 42**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2014.

Fourth Semester

Mathematics

MEASURE AND INTEGRATION

(For those who joined in July 2012 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Outer measure is not ————.
   (a) Countably sub-additive
   (b) Translation invariant
   (c) Defined for all sets of real numbers
   (d) Countably additive

2. Any ———— is measurable
   (a) Set (b) Subset of R
   (c) Countable set (d) Uncountable set

3. Let f be a non negative measurable function on E. Then $\int_E f = 0$ if and only if ———— on E.
   (a) $f = 0$ (b) $f = 0 \ a \cdot e$
   (c) $f =$ constant (d) $f =$ constant $a \cdot e$

4. Let the non negative function f be integrable over E. The f is ———— on E.
   (a) Finite $a \cdot e$ (b) Finite
   (c) Zero (d) Constant

5. Let E be a set of finite outer measure and F a collection of closed, bounded intervals that covers E in the sense of Vitali. Then for each $\in > 0$, there is a finite disjoint subcollection $\{I_k\}_{k=1}^{n}$ of F for which ————.
   (a) $m^*\left(\bigcup_{k=1}^{n} I_K\right) < \in$
   (b) $m^*\left(E \sim \bigcup_{k=1}^{n} I_K\right) < \in$
   (c) $m^*\left(\bigcup_{k=1}^{n} I_K \sim E\right) < \in$
   (d) $m^*\left(E \sim \bigcap_{k=1}^{n} I_K\right) < \in$

6. Let f be an increasing function on the closed, bounded interval $[a,b]$. Then for each $\alpha > 0, m^*\{x \in (a,b) \mid \overline{D}f(x) \geq \alpha\} \leq$ ________.

(a) $\frac{1}{\alpha}(f(a) - f(b))$

(b) $\frac{1}{a-b}f(\alpha)$

(c) $\frac{1}{b-a}f(\alpha)$

(d) $\frac{1}{\alpha}(f(b) - f(a))$.

7. The counting measure on an uncountable set is ________.

(a) σ-finite (b) not σ-finite

(c) σ-infinite (d) finite

8. Let (X, M, μ) be a measure space. If A and B are measurable sets, $A \subseteq B$ and $\mu(A) < \infty$, then $\mu(B \sim A) =$ ________.

(a) $\mu(B) - \mu(A)$ (b) $|\mu(B) - \mu(A)|$

(c) $\mu(A) - \mu(B)$ (d) $\mu(B)$

9. Let (X, M, μ) be a measure space and ψ non negative simple function on X. If $X_\circ \subseteq X$ is measurable and $\mu(X \sim X_\circ) = 0$, then $\int_X \psi \, d\mu$ = ________.

(a) 0

(b) $\int_{X_\circ} \psi \, d\mu$

(c) $\int_{X - X_\circ} \psi \, d\mu$

(d) $1 - \int_{X_0} \psi \, d\mu$

10. Let (X, M, μ) be a measure space and f a non negative measurable function on X. Then f is said to be integrable over X with respect to μ provided ________.

(a) $\int_X f \, d\mu = 0$ (b) $\int_X f \, d\mu = \infty$

(c) $\int_X f \, d\mu < \infty$ (d) $\int_X f \, d\mu = 1$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let f and g be measurable functions on E that are finite $a \cdot e \cdot$ on $E \cdot$ For any α and β, show that $\alpha f + \beta g$ is measurable on E.

Or

(b) Show that any set of outer measure zero is measurable and hence prove that any countable set is measurable.

12. (a) State and prove Lusin's theorem.

Or

(b) State and prove the monotone convergence theorem, on Lebesgue integration.

13. (a) Let f be a monotone function on (a,b). Prove that f is continuous except possibly at a countable number of points in (a,b).

Or

(b) Show that a function f on a closed, bounded interval [a,b] is absolutely continuous on [a,b] if any only if it is an indefinite integral over [a,b].

14. (a) Let (X, M, μ) be a measure space. If E_1 and E_2 are measurable and $\mu(E_1 \Delta E_2) = 0$, prove that $\mu(E_1) = \mu(E_2)$.

Or

(b) Let μ be a measure and μ_1 and μ_2 be mutually singular measures on a measurable space (X, M) for which $\mu = \mu_1 - \mu_2$. Show that $\mu_2 = 0$.

15. (a) State and prove the simple approximation lemma, on general measure space.

Or

(b) State and prove the Chebychev's inequality on General Measure space.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Show that the outer measure of an interval is its length.

Or

(b) Prove that every interval is measurable.

17. (a) State and prove the bounded convergence theorem.

Or

(b) State and prove Fatou's lemma.

18. (a) State and prove Lebesgue's theorem.

Or

(b) State and prove Jordan's theorem.

19. (a) State and prove Hahn's lemma.

Or

(b) State and prove the Caratheodory – Hahn theorem.

20. (a) (X, M, μ) is a measure space. Let E be a measurable subset of X and f an extended real-valued function on X. Show that f is measurable if and only if its restrictions to E and $X \sim E$ are measurable.

Or

(b) Consider two extended real-valued measurable functions f and g on X that are finite a.e. on X. Define X_0 to be the set of points in X at which f and g are finite. Show that X_0 is measurable and $\mu(X \sim X_0) = 0$.

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