(8 pages) **Reg. No. :** ....

Code No. : 7832 **Sub. Code : PMAM 12**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

First Semester

Mathematics – Core

ANALYSIS – I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The set of all subsequential limits of a sequence in a metric space X from a ———— subset of X.

   (a) open

   (b) closed

   (c) countable

   (d) perfect

2. A metric space is called separable if it contains a ———— dense subset.

   (a) countable (b) uncountable

   (c) perfect (d) none of these

3. $\lim\limits_{n\to\infty}\left[1+\frac{1}{n}\right]^n =$ ————.

   (a) 0 (b) 1

   (c) e (d) None of these

4. The series $\sum \frac{(-1)^n}{n}$ is ————.

   (a) converges

   (b) diverges

   (c) converges absolutely

   (d) none of these

5. If the series $\sum |a_n|$ converges then the series $\sum a_n$ is said to ————.

   (a) diverges

   (b) converges

   (c) converges absolutely

   (d) converges non absolutely

6. Let $\alpha = \lim_{n\to\infty} \sup \sqrt[n]{a_n}$ then $\sum a_n$ diverges if __________.

(a) $\alpha = 1$ (b) $\alpha < 1$

(c) $\alpha > 1$ (d) $\alpha = 0$

7. Let f be monotonic on (a, b) then the set of points of (a, b) at which f is discontinuous is __________.

(a) countable (b) uncountable

(c) atmost countable (d) none of these

8. Let f be a continuous mapping of a metric space X into a metric space Y then f is uniformly continuous on X if X is __________.

(a) connected (b) closed

(c) compact (d) none of these

9. Let f be defined by $f(x) = \begin{cases} x^2 \sin\frac{1}{x} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$

(a) f is differentiable at all points x

(b) f is differentiable at $x = 0$ and f is differentiable at other points

(c) f is not differentiable at all points x

(d) none of these

10. Suppose f is differentiable in (a, b) if __________ then f is monotonically increasing.

(a) $f'(x) = 0$

(b) $f'(x) \geq 0$

(c) $f'(x) \leq 0$

(d) None of these

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If E is an infinite subset of compact set K. Prove that E has a limit point in K.

Or

(b) Let K be a positive integer. If $\{I_n\}$ is a sequence of K -cells such that $I_n \supset I_{n+1}$, $n = 1, 2, 3, \ldots$ then prove that $\prod_{n=1}^{\infty} I_n$ is not empty.

12. (a) Prove that $\sum \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

Or

(b) Prove that e is irrational.

13. (a) State and prove Ratio test.

Or

(b) If $\sum a_n = A$ and $\sum b_n = B$ then prove that $\sum (a_n + b_n) = A + B$ and $\sum C a_n = CA$ for any fixed C.

14. (a) Prove that composition of continuous function is continuous.

Or

(b) Suppose f is a continuous mapping of a compact metric space X into a metric space Y, then prove that $f(X)$ is compact.

15. (a) Let f be defined by $f(x) = \begin{cases} x \sin \frac{1}{x} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$ prove that f is differentiable at all points except $x = 0$.

Or

(b) Let f be defined on $[a, b]$. If f is differentiable at a point $x \in [a, b]$ then prove that f is continuous at x.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove Heine Borel theorem.

Or

(b) Prove that Cantor set is a perfect set.

17. (a) Prove that :

(i) $\lim_{n \to \infty} \sqrt[n]{n} = 1$.

(ii) If $p > 0$ and α is a real then $\lim_{n \to \infty} \frac{n^{\alpha}}{(1+p)^n} = 0$.

Or

(b) Prove that if $\overline{E}$ is the closure of a set E in a metric space X then $diam\, \overline{E} = diam\, E$.

18. (a) Suppose :

(i) $\sum_{n=0}^{\infty} a_n$ converges absolutely

(ii) $\sum_{n=0}^{\infty} a_n = A$

(iii) $\sum_{n=0}^{\infty} b_n = B$

(iv) $C_n = \sum_{k=0}^{n} a_k b_{n-k} n = 0, 1, 2...$

then prove that $\sum_{n=0}^{\infty} C_n = AB$.

Or

(b) Prove that for any sequence $\{C_n\}$ of positive numbers, $\lim_{n\to\infty} \sqrt[n]{C_n} \le \lim_{n\to\infty} \sup \frac{C_{n+1}}{C_n}$.

19. (a) Let f be monotonically increasing on (a, b) then prove that $f(x+)$ and $f(x-)$ exists at every point of x of (a, b). More precisely, $\sup f(t) = f(x-) \le f(x) \le f(x+) = \inf f(t)$

$x < t < b$.

Or

(b) Let f be continuous mapping of a compact metric space X into a metric space Y then prove that f is uniformly continuous on X.

20. (a) State and prove Taylor's theorem.

Or

(b) State and prove Chain rule for differentiation.
