

(7 pages)

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M.Sc.(CBCS) DEGREE EXAMINATION,
NOVEMBER 2013.

Third Semester

Mathematics

TOPOLOGY

(For those who joined in July 2012 onwards)

Time : Three hours

Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer:

1. Let T_1 and T_2 be topologies on a set X such that $T_1 \subseteq T_2$. Then
 - (a) T_1 is finer than T_2
 - (b) T_2 is finer than T_1
 - (c) T_2 is coarser than T_1
 - (d) T_1 is incomparable with T_2

2. What is the maximum number of open sets in a topological space X having n points?
 - (a) n^2
 - (b) 2^n
 - (c) 2
 - (d) none of the above
3. The function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by
$$f(x) = \begin{cases} 0 & \text{if } -\infty < x < 0 \\ x & \text{if } 0 \leq x < 1 \\ 1 & \text{if } 1 \leq x < \infty \end{cases}$$
is discontinuous at
 - (a) 0 and 1
 - (b) all points of $\mathbb{R}$
 - (c) no point of $\mathbb{R}$
 - (d) at 1 only
4. Let $f : X \rightarrow Y$ be continuous and E be a closed subset of Y . Then
 - (a) $f^{-1}(E)$ is open
 - (b) $f^{-1}(E)$ need not be closed
 - (c) $f^{-1}(E)$ is closed
 - (d) none of the above
5. The subset $[0,1) \cup [1,2)$ of $\mathbb{R}$ with the usual topology is
 - (a) Connected
 - (b) compact
 - (c) not connected
 - (d) none of above



6. Which of the following is true?
- (a) Every compact set is closed
 - (b) Every closed set is compact
 - (c) Closed subset of a compact space is compact
 - (d) Any closed and bounded subset of a compact space is compact
7. $\mathbb{R}$ with usual topology is
- (a) Neither first countable nor second countable
 - (b) Both first countable and second countable
 - (c) Second countable but not first countable
 - (d) First countable but not second countable
8. Which of the following is true?
- (a) Every compact space is Hausdorff
 - (b) Every compact Hausdorff space is regular
 - (c) Every Hausdorff space is regular
 - (d) None of the above
9. Which of the following is true?
- (a) Every regular space is completely regular
 - (b) Every Hausdorff space is completely regular
 - (c) Every completely regular space is regular
 - (d) None of the above

10. A regular space is also known as

- (a) T_1
- (b) T_2
- (c) T_3
- (d) T_4

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Write down any five different topologies on the set $\{1,2,3,4,5\}$
- Or
- (b) Define order topology and show that the order topology on $\mathbb{Z}_+$, the set of all positive integers, is the discrete topology.
12. (a) Suppose $f : X \rightarrow Y$ is continuous and $A \subseteq X$. If a is a limit point of A , is it necessary that $f(a)$ is a limit point of $f(A)$. Prove your claim.
- Or
- (b) Find a function $f; \mathbb{R} \rightarrow \mathbb{R}$ which is continuous at exactly one point.
13. (a) If C and D form a separation of a topological space X , and if Y is a connected subset of X , prove that Y lies entirely in C or D .
- Or
- (b) Prove that every closed subset of a compact set is compact.



14. (a) Give an example of a space which does not satisfy the second countable axiom and justify your answer.

Or

- (b) Show that a closed subset of a normal space is normal.
15. (a) Prove that a subspace of a completely regular space is completely regular.

Or

- (b) State and prove imbedding theorem.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that the collection $B = \{(a, b) / a < b, a, b \in \mathbb{Q}\}$ is a countable basis for the usual topology of $\mathbb{R}$.

Or

- (B) Let $\{T_\alpha\}$ be a collection of topologies on a set X . Prove that there exists a largest topology contained in all T_α and there exists a smallest topology containing all T_α .

17. (a) Let $\{X_\alpha\}$ be a collection of topological spaces. Define the box topology and product topology on the Cartesian product $\prod X_\alpha$ and compare them.

Or

- (b) Explain the concept of quotient topology with a suitable examples.
18. (a) Prove that the product of two compact spaces is compact.

Or

- (b) (i) Prove that the union of connected subsets of a topological space is connected if they have a point in common.
- (ii) Prove that finite Cartesian product of connected spaces is connected.

19. (a) Prove the following :

- (i) The product of *Lindelöf* spaces need not be *Lindelöf*.
- (ii) A subspace of *Lindelöf* space need not be *Lindelöf*.

Or

- (b) Prove that every well ordered set with order topology is normal.



20. (a) State and prove Urysohn lemma.

Or

(b) State and prove Tietze extension theorem.

