(8 pages) **Reg. No. : ....**

Code No. : SS 30345 E Sub. Code : JAMA 21/ SAMA 21

B.Sc. (CBCS) DEGREE (Special Supplementary) EXAMINATION, APRIL 2020.

Second/Fourth Semester

Mathematics – Allied

VECTOR CALCULUS AND FOURIER SERIES

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer.

1. If $\bar{r} = x\vec{i} + y\vec{j} + z\vec{k}$ then $\nabla \times \bar{r}$ = ————.

(a) 0 (b) 1

(c) $\overline{0}$ (d) 3

2. The unit normal to the curved surface of the cylinder $x^2 + y^2 = 4; z = 0$ and $z = 3$ is

(a) $x\vec{i} + y\vec{j}$ (b) $\frac{x\vec{i} + y\vec{j}}{2}$

(c) $\frac{x\vec{i} + y\vec{j}}{3}$ (d) $\frac{x\vec{i} + y\vec{j}}{4}$

3. If D is the region bounded by $x = 0; x = 2; y = 0; y = 2$ then $\iint_D dx\,dy =$ ————.

(a) 2 (b) 1

(c) 4 (d) $\frac{1}{4}$

4. $\int_0^1\int_0^2\int_0^3 dx\,dy\,dz =$ ————

(a) 1 (b) 6

(c) 8 (d) 27

5. If $\bar{f} = x^2\vec{i} - xy\vec{j}$ and C is the straight line joining the points (0, 0) and (1, 1) then $\int_C \bar{f} \cdot d\bar{r} =$ ————.

(a) 1 (b) –1

(c) 2 (d) 0

6. If R is the projection of the surface S on the yz-plane then $\iint_S \bar{f} \cdot \bar{n}\, ds =$ ————————.

(a) $\iint_R \frac{\bar{f} \cdot \bar{n}}{\left|\bar{n} \cdot \vec{k}\right|} dx\, dz$ (b) $\iint_R \frac{\bar{f} \cdot \bar{n}}{\left|\bar{n} \cdot \vec{i}\right|} dy\, dz$

(c) $\int_C \bar{f} \cdot d\bar{r}$ (d) 0

7. If V is the volume enclosed by the closed surface S then the value of $\iint_S \bar{r} \cdot \bar{n} ds$ is ————————.

(a) $3V^2$ (b) $3\ V$

(c) $6\ V$ (d) 0

8. Gauss divergence theorem connects

(a) Line integral and double integral

(b) Line integral and surface integral

(c) Double integral and surface integral

(d) Surface integral and volume integral

9. The value of a_0 in the Fourier series expansion for the function $f(x) = x; -\pi < x < \pi$ is

(a) 0 (b) 1

(c) π (d) π^2

10. If $f(x)$ is an odd function in the interval $-\pi < x < \pi$ then

(a) $a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\, dx$; $b_n = 0$

(b) $a_n = 0$; $b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin x\, dx$

(c) $a_n = 0$; $b_n = 0$

(d) none of these

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $\bar{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\bar{r}|$ then prove that $\nabla \cdot (r^n \bar{r}) = (n+3) r^n$.

Or

(b) If $\nabla \varphi = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ then find the value of φ.

12. (a) Evaluate $\int_0^{\pi}\int_0^{a\cos\theta} r \sin\theta\, dr\, d\theta$.

Or

(b) Evaluate $\int_0^{a}\int_0^{x}\int_0^{y} xyz\, dz\, dy\, dx$.

13. (a) Evaluate $\int_C \overline{F} \cdot d\overline{r}$ if $\overline{F} = 3xy\vec{i} - 4z\vec{j} + 10x\vec{k}$ along the curve $x = t_{+1}^{2}$; $y = 2t^2$; $z = t^3$ from $t = 0$ to $t = 2$.

Or

(b) Evaluate $\int_S \overline{A} \cdot \overline{n}\, ds$ where $\overline{A} = 18z\vec{i} - 12\vec{j} + 3y\vec{k}$ and S is the part of the plane $2x + 3y + 6z = 12$ in the first octant.

14. (a) Using Green's theorem, evaluate $\int_C (xy - x^2)\,dx + x^2 y\, dy$ along the closed curve C formed by $x = 1$; $y = 0$ and $y = x$.

Or

(b) If $\bar{f} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ and V is the volume enclosed by the cube $0 \le x, y, z \le 1$ then evaluate $\iiint_V \nabla \cdot \bar{f}\; dV$.

15. (a) Find the cosine series for $f\,x) = x$ in $(0, \pi)$.

Or

(b) Find the Fourier series of

$$f(x) = \begin{cases} -1 \; if \; -\pi < x \le 0 \\ \;\;1 \; if \; 0 \le x < \pi. \end{cases}$$

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) (i) Find the directional derivative of $\varphi = x + xy^2 + yz^2$ at (0, 1, 1).

(ii) Prove that $div\,(\bar{f} \times \bar{g}) = \bar{g} \cdot curl\,\bar{f} - \bar{f} \cdot curl\,\bar{g}$.

Or

(b) (i) If $\bar{f}$ is solenoidal then prove that curl curl curl curl $\bar{f} = \nabla^4 \bar{f}$.

(ii) Find the value of the constant 'a' so that the vector $\bar{f} = (axy - z^3)\vec{i} + (a-2)x^2\vec{j} + (1-a)xz^2\vec{k}$ will be irrotational.

17. (a) Evaluate $\iint\limits_D (1 + x + y)\,dx\,dy$ where D is the region bounded by $y + x = 0$; $x = \sqrt{y}$; $y = 0$ and $y = 1$.

Or

(b) Using triple integral, find the volume of the sphere $x^2 + y^2 + z^2 = a^2$.

18. (a) Evaluate $\iint_S \bar{f} \cdot \bar{n}\, ds$ where $\bar{f} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ and S is the surface of the cube bounded by $x = 0$; $x = 1$; $y = 0$; $y = 1$; $z = 0$ and $z = 1$.

Or

(b) Evaluate $\int_C \bar{f} \cdot d\bar{r}$ where $\bar{f} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ and C is the rectangle in the xy-plane bounded by $x = 0$; $x = a$; $y = 0$; $y = b$.

19. (a) Verify Gauss divergence theorem for $\bar{f} = y\vec{i} + x\vec{j} + z^2\vec{k}$ and the cylindrical region S given by $x^2y^2 = a^2$; $z = 0$ and $z = 4$.

Or

(b) Verify Stoke's theorem for $\bar{f} = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

20. (a) Show that $x^2 = \frac{\pi}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n \cos nx}{n^2}$ in the interval $-\pi \le x \le \pi$ and deduce that

(i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$

(ii) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots = \frac{\pi^2}{12}$.

Or

(b) Find a sine series in the range 0 to π for the function $f(x) = \begin{cases} x; & 0 < x < \frac{\pi}{2} \\ \pi - x; & \frac{\pi}{2} < x < \pi. \end{cases}$