(7 pages) **Reg. No. :** .....

Code No. : 9359 **Sub. Code : HMAM 23**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2015.

Second Semester

Mathematics

CLASSICAL MECHANICS

(For those who joined in July 2012 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions

Choose the correct answer :

1. The moment of a force about a point is given by

   (a) $\overline{L} = \overline{r} \times \overline{p}$ (b) $N = \overline{r} \times \overline{F}$

   (c) $\overline{F} = m\overline{a}$ (d) $\overline{P} = m\frac{d\overline{R}}{dt}$

2. If a bead is sliding on a rigid curved wire fixed in space, the constraint is

   (a) Holonomic (b) Non-holonomic

   (c) Rheonomous (d) Scleronomous

3. The equation $\sum_i \left(F_i^{(a)} - \dot{p}_i\right)\delta r_i = 0$ is called

   (a) The principle of virtual work

   (b) D'Alembert's principle

   (c) Hamilton's principle

   (d) Fermat's principle

4. The equation $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0$ is called

   (a) Lagrange equations

   (b) Hamilton-Jacobi equations

   (c) Hamilton's equations

   (d) Energy equations

5. The motion of the system from time t_1 to time t_2 such that the line integral $I = \int_{t_1}^{t_2} L\,dt$, where $L = T - V$, has a stationary value for the correct path of the motion. This is called

   (a) The principle of virtual work

   (b) D' Alembert's principle

   (c) Hamilton's principle

   (d) Fermat's principle

6. A surface of revolution is formed by taking some curve passing between two fixed end points and revolving at about y-axis. The curve for which surface area is minimum is

(a) Straight line (b) Catenary

(c) Cardioid (d) Cycloid

7. Which of the following theorem is used in deriving Boyle's law for perfect gases?

(a) Conservation theorem for linear momentum

(b) Conservation theorem angular momentum

(c) Virial theorem

(d) Cartheodory theorem

8. If $e > 1$ and $E > 0$, then the orbit is

(a) Circle (b) Ellipse

(c) Parabola (d) Hyperbola

9. The radius vector sweeps out equal areas in equal times. This is known as

(a) Newton second law of motion

(b) Kepler's first law of planetary motion

(c) Kepler's second law of planetary motion

(d) Kepler's third law of planetary motion

10. If r_1 and r_2 are apsidal distances, then the semi-major axis is given by

(a) $r_1 - r_2$ (b) $r_1 + r_2$

(c) $\frac{r_1 + r_2}{2}$ (d) $\frac{r_1 - r_2}{2}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that for a single particle with constant mass the equation of motion implies the following differential equation for the kinetic energy $\frac{dT}{dt} = \overline{F}.\overline{v}$ while if the mass v aries with time the corresponding equation is $\frac{d}{dt}(mT) = \overline{F}.\overline{p}$.

Or

(b) Two points of mass m are joined by a rigid weightless rod of length l, the centre of which is constrained to move on a circle of radius a. Set up the kinetic energy in generalized coordinates.

12. (a) State and prove the principle of virtual work.

Or

(b) Find the equation of motion of the Atwood's machine.

13. (a) Prove that the shortest distance between two points in plane is a straight line.

Or

(b) Define cyclic coordinate. Prove that the generalized momentum conjugate to a cyclic coordinate is conserved

14. (a) Prove that the central force motion of two bodies about their centre of mass can always be reduced to an equivalent one-body problem.

Or

(b) With usual notation, derive the equation $\overline{T} = \frac{(n+1)}{2}\overline{V}$.

15. (a) State and Prove Kepler's third law of planetary motion.

Or

(b) With usual notation prove that $\tan\frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{\psi}{2}$

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) State and prove conservation theorem for total angular momentum of a system of particles.

Or

(b) State and prove total energy conservation theorem for a system of particles.

17. (a) Prove that the kinetic energy of a system can always be written as the sum of three homogeneous functions of velocities.

Or

(b) Find the equation of motion of a particle using plane polar coordinates.

18. (a) A surface of revolution is formed by taking some curve passing between two fixed end points (x_1, y_1) and (x_2, y_2) and revolving it about y – axis. Find the curve for which the surface area is minimum.

Or

(b) Derive Euler-Lagrange differential equation.

19. (a) Derive the differential equation for the orbit if the force law or the potential is known.

Or

(b) With usual notation, prove that $\frac{1}{2}m\dot{r}^2 + \frac{1}{2}\frac{l^2}{mr^2} + V = \text{constant}$.

20. (a) For circular and parabolic orbits in an attractive $\frac{1}{r}$ potential having the same angular momentum, show that

(i) Perihelion distance of the parabola is one half the radius of the circle.

(ii) The speed of the particle at any point in a parabolic orbit is $\sqrt{2}$ times the speed in a circular orbit passing through the same point.

Or

(b) Evaluate approximately the ratio of the mass of the sun to that of the earth using length of the year 365 ¼ days, lunar month 27.3 days, mean radii of earth's orbit 1.49×10^8 km and of the moon's orbit 3.8×10^5 km.
