(6 pages) **Reg. No. :** ....

Code No. : 10294 E **Sub. Code : AMMA 61**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Sixth Semester

Mathematics — Core

COMPLEX ANALYSIS

(For those who joined in July 2020 only)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. $f(z) = \frac{\bar{z}}{z}$ has ———— limit as $z \to 0$.

   (a) 0 (b) 1

   (c) ∞ (d) no

2. The function $f(z) = \sqrt{|xy|}$ is ————.

   (a) not differentiable at $z = 0$

   (b) differentiable at every point

   (c) nowhere differentiable

   (d) nowhere continuous

3. If C is the circle, then $\int_C \frac{dz}{z-a}$ is ————.

   (a) 0 (b) πi

   (c) $2\pi i$ (d) $4\pi i$

4. If C is $|z| = 4$, then the value of $\frac{1}{2\pi i}\int_C \frac{z^2+5}{z-3}dz$ ————.

   (a) 12 (b) 14

   (c) 8 (d) 0

5. For $f(z) = z^2 \sin z$, $z = 0$ is a zero of order ————.

   (a) 0 (b) 1

   (c) 2 (d) 3

6. The residue of $\frac{e^z}{z^2}$ at $z = 0$ is ————.

   (a) 0 (b) 1

   (c) 2 (d) 3

7. $\frac{z + z^{-1}}{2} =$ ————.

   (a) 0 (b) z

   (c) $\sin\theta$ (d) $\cos\theta$

8. The simple poles of $f(z) = \frac{2}{(z+5i)(z-5i)}$ is

(a) $\frac{-i}{5}, -i$ (b) $\frac{-i}{5}, -5i$

(c) $-i, -5i$ (d) $-i, \frac{-i}{5}$

9. A bilinear transformation with only one finite fixed point is called ————.

(a) Hyperbolic (b) Parabolic

(c) Elliptic (d) Line

10. The fixed point of the transformation $w = \frac{1}{z-2i}$.

(a) 1 (b) -1

(c) i (d) $-i$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $\lim_{z\to 2} \frac{z^2-4}{z-2} = 4$.

Or

(b) Show that an analytic function in a region with constant modulus is constant.

12. (a) Evaluate the integral $\int_C (x^2 - iy^2)\, dz$ where C is the parabola $y = 2x^2$ from (1, 2) to (2, 8).

Or

(b) State and prove Cauchy's inequality.

13. (a) Expand $\frac{-1}{(z-1)(z-2)}$ as a power series in z in the region $|z| < 1$.

Or

(b) State and prove Cauchy's residue theorem.

14. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta}$.

Or

(b) Prove that $\int_0^{\infty} \frac{\cos x}{1+x^2}\, dx = \frac{\pi}{2e}$.

15. (a) Find the image of the circle $|z - 3i| = 3$ under the map $w = \frac{1}{z}$.

Or

(b) Find the bilinear transformation from $z = 0, -i, -1$ to $w = i, 1, 0$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Derive $C-R$ equations in polar form.

Or

(b) If $f(z)=u+iv$ is an analytic function and $u(x, y)=\dfrac{\sin 2x}{\cos h\,2y+\cos 2x}$, find $f(z)$.

17. (a) State and prove Cauchy's theorem.

Or

(b) State and prove Cauchy's integral formula.

18. (a) State and prove Laurent's theorem.

Or

(b) State and prove Rouche's theorem.

19. (a) Evaluate $\int_0^{2\pi} \dfrac{d\theta}{2+\cos\theta}$.

Or

(b) Prove that $\int_0^{\infty} \dfrac{dx}{x^6+1}=\dfrac{\pi}{3}$.

20. (a) Find the bilinear transformation having two invariant points α and ∞ where α is finite.

Or

(b) Prove that any bilinear transformation can be expressed as a product of translation, rotation, magnification or contraction and inversion.
