(6 pages) **Reg. No. :** .....

Code No. : 6370 **Sub. Code : HMAE 41**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2017.

Fourth Semester

Mathematics

Elective — GRAPH THEORY

(For those who joined in July 2012-2015)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The number of edges of $K_{7,4}$ is

   (a) 28 (b) 11

   (c) 7 (d) 3

2. If G is simple, then

   (a) $\epsilon \leq \binom{r}{2}$ (b) $\epsilon = \binom{r}{2}$

   (c) $\epsilon < \binom{r}{2}$ (d) $\gamma \leq \binom{\epsilon}{2}$

3. A connected graph that has no cut vertices is called a

   (a) Complete graph (b) Hamilton graph

   (c) Block (d) Maximal matching

4. The Petersen graph is

   (a) Both Hamiltonian and Eulerian

   (b) Neither Hamiltonian not Eulerian

   (c) Hamiltonian but not Eulerian

   (d) Non Hamiltonian but Eulerian

5. G has a perfect matching if and only if $O(G-S) \leq |S|$ for all $S \subset V$. This theorem is known as

   (a) Konig's theorem (b) Bondy's theorem

   (c) Tutte's theorem (d) Hall's theorem

6. If G is bipartite, then

   (a) $\delta = \Delta$

   (b) $\chi' = \delta$

   (c) $\chi' = \Delta$

   (d) $\chi' = \Delta$ or $\Delta + 1$

7. The number of vertices in a maximum covering of G is denoted by

(a) α (b) β

(c) α' (d) β'

8. The value of $r(2,5)$ is

(a) 10 (b) 7

(c) 5 (d) 2

9. Every ———— graph is a block

(a) Chromatic (b) Critical

(c) Connected (d) Simple

10. $\pi_k(\lhd)$ is

(a) k^3+6k^2+2 (b) $k(k-1)(k-2)$

(c) k^3 (d) k^3+4k+5

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, by choosing either (a) or (b).

11. (a) Show that if G is simple and $\varepsilon > \binom{\gamma-1}{2}$, then G is connected.

Or

(b) If G is a tree, prove that $\varepsilon = \gamma - 1$.

12. (a) Show that a connected graph has an Euler trail if and only if it has atmost two vertices of odd degree.

Or

(b) Show that $C(G)$ is well defined.

13. (a) Show that every 3 – regular graph with out cut edges has a perfect matching.

Or

(b) Let G be a connected graph that is not an odd cycle. Prove that G has a 2-edge colouring in which both colours are represented at each vertex of degree at least two.

14. (a) Prove that a set $S \subseteq V$ is an independent set of G if and only if $V - S$ is a covering of G and hence show that $\alpha + \beta = \gamma$.

Or

(b) For any two integers $k \geq 2$ and $l \geq 2$, prove that $r(k,l) \leq r(k,l-1) + r(k-1,l)$.

15. (a) In a critical graph, prove that no vertex cut is a clique.

Or

(b) If G is simple, prove that $\pi_k(G) = \pi_k(G-e) - \pi_k(G \cdot e)$ for any edge e of G.

SECTION C — (5 × 8 = 40 marks)

Answer ALL questions, by choosing either (a) or (b).

16. (a) Obtain a characterisation of bipartite graphs.

Or

(b) Let G be a graph with $\gamma - 1$ edges. Show that the following three statements are equivalent

(i) G is connected

(ii) G is a tree

(iii) G is a cyclic.

17. (a) Explain the notations k, k' and δ obtain the inequality connecting k, k' and δ.

Or

(b) Show that a nonempty connected graph is Eulerian if and only if it has no vertices of odd degree.

18. (a) State and prove Hall's theorem.

Or

(b) If G is simple, prove that $\chi' = \Delta$ or $\chi' = \Delta + 1$.

19. (a) If $\delta > 0$, prove that $\alpha' + \beta' = \gamma$.

Or

(b) Prove that $r(k,k) \geq 2^{k/2}$.

20. (a) If G is 4-chromatic, prove that G a contains a subdivision of K_4.

Or

(b) For any positive integer k, prove that there exists a k-chromatic graph containing no triangle.

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