(6 pages) **Reg. No. :** ....

Code No. : 40335 E **Sub. Code : JMMA 31/ JMMC 31/SMMA 31**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Third Semester

Mathematics/ Mathematics with CA — Main

REAL ANALYSIS — I

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If $x > y$ and $y > z$ then

   (a) $x = z$ (b) $z > x$

   (c) $x > z$ (d) $y > x$.

2. If a and b are real, then $|a+b| \geq$

   (a) $|a| + |b|$ (b) $|a| - |b|$

   (c) $||a| - |b||$ (d) $|b| - |a|$.

3. The range of the sequence $(1 + (-1)^n)$ is

   (a) N (b) Z

   (c) $\{0, 1\}$ (d) $\{0, 2\}$.

4. $\lim\limits_{n\to\infty} \dfrac{2n+1}{2n} =$

   (a) 0 (b) 1

   (c) 2 (d) -1.

5. $\lim\limits_{n\to\infty} \dfrac{1}{n}\left(1 + \dfrac{1}{2} + \cdots + \dfrac{1}{n}\right) =$

   (a) 0 (b) e

   (c) 1 (d) ∞.

6. If $(a_n) \to a$ and $(b_n) \to b$ then

   (a) $(a_n + b_n) \to a + b$ (b) $(a_n - b_n) \to a - b$

   (c) $(a_n / b_n) \to a/b$ (d) $(a_n) + (b_n) \to a + b$.

7. Let Σa_n be a series of positive terms. Then $n \to \infty$ is

   (a) convergent if $\lim\limits_{n\to\infty} a_n^{1/n} > 1$

   (b) convergent of $\lim\limits_{n\to\infty} a_n^{1/n} < 1$

   (c) divergent if $\lim\limits_{n\to\infty} a_n^{1/n} < 1$

   (d) divergent if $\lim\limits_{n\to\infty} a_n^{1/n} = 1$.

8. If $a_n = \frac{n!}{n^n}$ then $\lim_{n\to\infty} \frac{a_n}{a_{n+1}} =$

(a) e (b) 1

(c) 0 (d) $1/e$.

9. The series $\sum \frac{(-1)^{n+1} n}{5n+1}$

(a) converges (b) diverges

(c) oscillates (d) both (a) and (c).

10. For the geometric series Σx^n the radius of convergence R is

(a) 0 (b) 1

(c) ∞ (d) $1/n$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Write down the order axioms.

Or

(b) State and prove triangle inequality.

12. (a) Prove that any convergent sequence is bounded sequence.

Or

(b) If $(a_n) \to l$, $(b_n) \to l$ and $a_n \le c_n \le b_n$ for all n, then prove that $(c_n) \to l$.

13. (a) State and prove Cesaro's theorem.

Or

(b) Prove that every sequence (a_n) has a monotonic sequence.

14. (a) Discuss the convergence of the series $1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \cdots$

Or

(b) State and prove Raabe's test.

15. (a) Show that the series

$$\frac{1}{2^3} - \frac{1}{3^3}(1+2) + \frac{1}{4^3}(1+2+3) - \frac{1}{5^3}(1+2+3+4) + \cdots$$

converges.

Or

(b) Find the radius of convergence, for the binomial series.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove Additive property.

Or

(b) State and prove Cauchy-Schwarz inequality.

17. (a) If $(a_n) \to a$ and $a_n \neq 0$ for all n and $a \neq 0$, then prove that $\left(\frac{1}{a_n}\right) \to \left(\frac{1}{a}\right)$.

Or

(b) Show that $\lim_{n\to\infty} \left(a^{1/n}\right) = 1$, where $a > 0$ is any real number.

18. (a) Discuss the convergence of the geometric sequence (r^n).

Or

(b) Prove that

$$\frac{1}{n}\left[(n+1)(n+2)\cdots(n+n)\right]^{1/n} \to 4/e.$$

19. (a) Prove that the harmonic series $\sum \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

Or

(b) State and prove Kummer's test.

20. (a) Show that the series $\sum \frac{\sin n\theta}{n}$ converges for all values of θ and $\sum \frac{\cos n\theta}{n}$ converges if θ is not a multiple of 2π.

Or

(b) State and prove the Abel's theorem.

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