(7 pages) **Reg. No. :** ....

Code No. : 8525 **Sub. Code : HMAM 24**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2013.

Second Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

(For those who joined in July 2012 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. In the general second order linear differential equation $y''+Py'+Qy=R$, P,Q and R are functions of ————

   (a) x alone (b) y alone

   (c) both x and y (d) None of these

2. $W(y_1, y_2)=$ ————

   (a) $y_1y_2-y_2y_1'$ (b) $y_1y_2'-y_2y_1'$

   (c) $y_2y_1'-y_1y_2'$ (d) None of these

3. The solution of $y'=2xy$ is $y=$ ————

   (a) ce^{-x^2} (b) ce^{x^2}

   (c) $ce^{x^2/2}$ (d) $ce^{-x^2/2}$

4. Any point that is not an ordinary point of $y''+P(x)y'+Q(x)y=0$ is called ———— point

   (a) a singular (b) regular singular

   (c) regular (d) special

5. The regular singular point of $x^2y''+xy'+(x^2-p^2)y=0$ is ————

   (a) 1 (b) –1

   (c) 0 (d) None of these

6. The indicial equation of the differential equation $2x^2y''+x(2x+1)y'-y=0$ is ————

   (a) $m(m-1)+\frac{m}{2}+\frac{1}{2}$

   (b) $m(m-1)+\frac{m}{2}-\frac{1}{2}$

   (c) $m(m-1)-\frac{m}{2}-\frac{1}{2}$

   (d) None

7. $P_n(-1) =$ ________

(a) -1 (b) 1

(c) $(-1)^n$ (d) $(-1)^2$

8. $J_{1/2}(x) =$ ________

(a) $\sqrt{\frac{2}{\pi x}}$ (b) $\sqrt{\frac{1}{\pi x}} \sin x$

(c) $\sqrt{\frac{2}{\pi x}} \sin x$ (d) $\sqrt{\frac{2}{\pi x}} \cos x$

9. The vanishing or non vanishing of the Wronskian $w(t)$ of two solutions ________ on the choice of t

(a) depends

(b) does not depend

(c) none

(d) either (a) or (b)

10. The exact solution of the initial value problem $y' = y^2$, $y(0) = 1$ is $y =$ ________

(a) $\frac{1}{1+x}$ (b) $\frac{1}{x-1}$

(c) $\frac{1}{1-x}$ (d) None

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that any linear combination of two solutions of the homogeneous equation $y'' + P(x)y' + Q(x)y = 0$ is also a solution.

Or

(b) Find a particular solution of $y'' + 4y = \tan 2x$.

12. (a) Find the general solution of $y'' + y = 0$ in the form $y = a_0 y_1(x) + a_1 y_2(x)$, where $y_1(x)$ and $y_2(x)$ are power series.

Or

(b) Find the general solution of $y'' + xy = 0$ in the form of $y = a_0 y_1(x) + a_1 y_2(x)$.

13. (a) Determine the nature of the point $x = 0$ for the following equation $y'' + (\sin x)y = 0$.

Or

(b) Show that $\int_{-\infty}^{\infty} W_m W_n \, dx = 0$ if $m \neq n$ where W_m, W_n are Hermite function of order m and n respectively.

14. (a) Find $P_0(x), P_1(x), P_2(x)$ and $P_3(x)$ using rodrigues formula where $P_n(x)$ is the n^{th} legendre polynomial.

Or

(b) Express $J_3(x)$ in terms of $J_0(x)$ and $J_1(x)$.

15. (a) Find the general solution of $\frac{dx}{dt} = -3 + 4y$; $\frac{dy}{dt} = -2x + 3y$.

Or

(b) Find the exact solution of the initial value problem $y' = y^2$, $y(0) = 1$ using Picard's method of successive approximation.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Show that $y = c_1 \sin x + c_2 \cos x$ is the general solution of $y'' + y = 0$ on any interval, and find the particular solution for which $y(0) = 2$ and $y'(0) = 3$.

Or

(b) Find the general solution of $(x^2 + x)y'' + (2 - x^2)y' - (2 + x)y = x(x+1)^2$.

17. (a) Find a power series solution of the form $\Sigma a_n x^n$ for the differential equation $xy' = y$. Also solve the equation directly and explain any discrepancies that arise.

Or

(b) Verify that the equation $y'' + y' - xy = 0$ has a three term recursion formula and find its series solution $y_1(x)$ and $y_2(x)$ such that $y_1(0) = 1$, $y_1'(0) = 0$, $y_2(0) = 0$, $y_2'(0) = 1$.

18. (a) Consider the differential equation $y'' + \frac{1}{x^2} y' - \frac{1}{x^3} y = 0$.

(i) Show that $x = 0$ is an irregular singular point.

(ii) Use the fact that $y_1 = x$ is a solution to find a second independent solution y_2.

Or

(b) Calculate two independent Frobenius series solutions for the differential equation $2x^2 y'' + xy' - (x+1)y = 0$.

19. (a) If $f(x) = x^p$ for the interval $0 \le x < 1$, show that its Bessel series in the functions $J_p(\lambda_n, x)$, where the λ_n's are the positive zeros of $J_p(x)$, is $x^p = \sum_{n=1}^{\infty} \frac{2}{\lambda_n J_{p+1}(\lambda_n)} J_p(\lambda_n x)$.

Or

(b) Show that the general solution of $\frac{dy}{dx} = x^2 + y^2$ can be written as

$$y = x \frac{J_{-3/4}\left(\frac{1}{2}x^2\right) + CJ_{3/4}\left(\frac{1}{2}x^2\right)}{CJ_{-1/4}\left(\frac{1}{2}x^2\right) - J_{1/4}\left(\frac{1}{2}x^2\right)}$$

20. (a) Show that $x = e^{4t}$, $y = e^{4t}$ and $x = e^{-2t}$, $y = e^{-2t}$ are solutions of the homogeneous system $\frac{dx}{dt} = x + 3y$, $\frac{dy}{dt} = 3x + y$. Also find the particular solution $x = x(t)$, $y = y(t)$ of this system for which $x(0) = 5$ and $y(0) = 1$.

Or

(b) Show that $f(x, y) = x^2|y|$ satisfies a Lipschitz condition on the rectangle $|x| \le 1$ and $|y| \le 1$ but that $\frac{\partial f}{\partial y}$ fails to exist at many points of this rectangle.