(6 Pages)

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Code No. : 5331 **Sub. Code : PMAM 42**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2022

Fourth Semester

Mathematics — Core

COMPLEX ANALYSIS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. A function u which satisfied the Laplace's equation $\Delta^2 u = 0$ is called as ————
   (a) harmonic (b) analytic
   (c) compact (d) empty

2. A series of the form $a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n + \cdots$ s called as ————
   (a) sequence (b) power series
   (c) converge (d) null

3. The linear transformation $w = kz$ is a ———— if = 1.
   (a) parallel (b) regular
   (c) rotation (d) inverse

4. The points z and z^* are said to be ———— with respect to the circle C through z_1, z_2, z_3 if and only if $(z^*, z_1, z_2, z_3) =$
   (a) reflexive (b) transitive
   (c) none (d) symmetric

5. Every Jordan curve in the plane determines exactly ———— regions.
   (a) two (b) one
   (c) zero (d) three

6. $n(\gamma, -a) =$ ————
   (a) $-n(\gamma, a)$ (b) $-n(\gamma, a)$
   (c) $n(\gamma, -a)$ (d) $-n(\gamma, -a)$

7. A function which is analytic and bounded in the whole plane must reduce to a ————
   (a) 0 (b) variable
   (c) constant (d) identity

8. If $f(a)=0$, there exists a first derivative $f^{(h)}(a)$ which is different from zero then 'a' is a zero of order ——————

(a) 1 (b) $h+1$

(c) $h-1$ (d) h

9. The residue of $\frac{e^2}{(z-a)(z-b)}$ at 'b' is ——————

(a) $\frac{e^a}{z-b}$ (b) 1

(c) ∞ (d) none

10. The poles and residues of is ——————

(a) –3, –2 and 1, 1 (b) –3, –2 and –1, 1

(c) 1, 2 and 2, 3 (d) 2, 3 and –1, 1

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Expand $\frac{2W+5}{W+2}$ in power series of $z-1$. What is the radius of convergence?

Or

(b) Prove that if all zeros of a polynomial $P(z)$ lies in a half plane, then all the zeros of the derivatives $P'(Z)$ lie in the same half plane.

12. (a) Prove that $\int_{-\gamma} f(z)\,dz = -\int_{\gamma} f(z)\,dz$.

Or

(b) If z_1, z_2, z_3, z_4 are distinct point in the extended plane and T any linear transformation, then prove that $(Tz_1, Tz_2, Tz_3, Tz_4) = (z_1, z_2, z_3, z_4)$.

13. (a) Prove that if the piecewise differentiable closed curve does not pass through the point a, then the value of the integral is a multiple of $2i$.

Or

(b) Suppose that $f(z)$ is analytic in an open disk and let γ be a closed curve in υ. Then prove that for any point 'a' not on γ, $\int_{\gamma} \frac{f(z)}{(z-a)} dz = 2\pi i n(\gamma, a)$ where n is the index of 'a' with respect to γ.

14. (a) Prove that if $f(z)$ is analytic and non constant in a region, then its absolute value has no maximum.

Or

(b) State and prove Taylor's theorem.

[P.T.O]

15. (a) Evaluate $\int_{|z|=1}^{-1} \frac{dz}{x^2+2az+1}$, $a>1$.

Or

(b) State and prove Rouche's theorem.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Derive Cacuhy Riemann equation in Cartesian coordinates.

Or

(b) State and prove Abel's limit theorem.

17. (a) Prove that the cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or on a straight line.

Or

(b) Prove that the line integral defined in depends only on the end points of if and only if there exists a function $U(x,y)$ in with the partial derivatives $p' = p$ and $q' = q$.

18. (a) Prove that if the function $f(z)$ is analytic on R, then $\int_{\partial R_n} z\,dz = 0$.

Or

(b) Let $f(z)$ be analytic on the set R' obtained from a rectangle R by omitting a finite number of interior points j if $f(z)=0$ for all j then prove that $\left|\int_{\partial R} f dz\right| < 8\varepsilon$.

19. (a) Suppose that is continuous on the arc. Then prove that the function $F_n(z)=0$ is analytic in each of the regions determined by and its derivative is $F_n'(z) = nF_{n+1}(z)$.

Or

(b) State and prove Cauchy's theorem.

20. (a) State and prove Cauchy residue theorem.

Or

(b) Evaluate by the method of residues $\int_0^{\infty} \frac{x^2 dx}{x^4+5x^2+6}$.