(8 pages) **Reg. No. :** ....

Code No. : 7753 **Sub. Code : WMAM 13/ VMAC 13**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2024.

First Semester

Mathematics – Core

ORDINARY DIFFERENTIAL EQUATIONS

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — (15 × 1 = 15 marks)

Answer ALL questions.

Choose the correct answer :

1. The characteristic polynomial of $y'' + y' - 2y = 0$ is ————

   (a) $r^2 + r + 2$ (b) $r^2 + r - 2$

   (c) $(r^2 + r - 2)y = 0$ (d) r_1, r_2

2. Two solutions φ_1, φ_2 of $L(y) = 0$ are linearly dependent iff $W(\varphi_1 \varphi_2) =$ ————

   (a) 0 (b) 1

   (c) ∞ (d) $\neq 0$

3. The value of the Wronksian is ———— if $\varphi_1 = \cos x$ and $\varphi_2 = \sin x$.

   (a) 0 (b) –1

   (c) 1 (d) 2

4. The roots of the characteristic polynomial of the equation $y''' - y' = 0$ are ————

   (a) 0, 1, –1 (b) 0, 1, 1

   (c) 1, 1, 1 (d) 1, –1, –1

5. The real valued solution of $y'' + y = 0$ is ————

   (a) $c_1 + c_2 x$ (b) $c_1 e^x + c_2 e^{-x}$

   (c) $c_1 \cos x + c_2 \sin x$ (d) $c_1 x + c_2 x^{-1}$

6. The particular solution of the equation $y'' + 4y = \cos x$ is ————

   (a) $\frac{1}{3} \sin x$ (b) $\frac{1}{3} x$

   (c) $\frac{1}{3}$ (d) $\frac{1}{3} \cos x$

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7. The linear differential equation $L(y) = b(x)$ is said to be non-homogeneous equation if $b(x)$ ———

(a) $= 0$ (b) $\neq 0$

(c) > 0 (d) < 0

8. The n functions $\varphi_1, \varphi_2, \ldots \varphi_n$ defined on an interval I are said to be ——— if the only constants $c_1 \ldots c_n$ such that $c_1\varphi_1(x) + \ldots + c_n\varphi_n(x) = 0$ for all x in I are the constants $c_1 = c_2 = \ldots = c_n = 0$.

(a) linearly independent

(b) linearly dependent

(c) cannot say

(d) both (a) and (b)

9. The value of the Legendre polynomial $p_1(x)$ is ———

(a) 1 (b) 0

(c) x (d) x^2

10. The singular point and its nature of the equation $x^2y'' - 5y' + 3x^2y = 0$ is ———

(a) $x = 0$, regular (b) $x = 0$, not regular

(c) $x = 1$, not regular (d) No singular point

11. A point x_0 such that $a_0(x_0) = 0$ is called ——— point of the equation $a_0(x)y^{(n-1)} + \ldots a_n(x)y = 0$.

(a) regular (b) particular

(c) not regular (d) singular

12. The origin $x_0 = 0$ is ——— for the equation $x^2y'' - y' - \frac{3}{4}y = 0$.

(a) singular point (b) regular singular

(c) irregular singular (d) analytic

13. The solution of $y' = y^2$ with $\varphi(1) = -1$ is ———

(a) $\frac{1}{x}$ (b) x

(c) x^2 (d) $-\frac{1}{x}$

14. The equation $(x^2 + xy)dx + xy\,dy = 0$ is ———

(a) exact (b) not exact

(c) can't say (d) both (a) and (b)

15. The Lipschitz constant for the function $f(x, y) = 4x^2 + y^2ons$, $|x| \leq 1$, $|y| \leq 1$

(a) 2 (b) 1

(c) 4 (d) 3

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PART B — ($5 \times 4 = 20$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If φ_1, φ_2 are two solutions of $L(y)=0$ on an interval I containing a point x_0 then prove that $W(\varphi_1, \varphi_2) = e^{-a_1(x-x_0)}\, W(\varphi_1, \varphi_2)(x_0)$.

Or

(b) Compute the solution of the initial value problem $y'' - 2y' - 3y = 0$; $y(0)=0$, $y'(0)=1$.

17. (a) Let $\varphi_1, \varphi_2, ...\varphi_n$ be linearly independent solution of $L(y)=0$ on an interval I if $c_1,....c_n$ are any constants then prove that $\varphi = c_1\varphi_1 + ... + \varphi_n c_n$ is a solution.

Or

(b) Find the solutions of $y''' - y' = x$.

18. (a) Find the solution φ for the equation $y'' - \frac{2}{x^2} y = x \; (0 < x < \infty)$. Given $\varphi_1 = x^2$, $\varphi_2 = x^{-1}$.

Or

(b) Verify $\varphi_1 = x^3$ satisfy the equation $x^2 y'' - 7xy' + 15y = 0 \quad (x > 0)$ and find $\varphi_2 = (x)$.

19. (a) Prove that $J_0'(x) = -J_1(x)$.

Or

(b) Calculate the roots of the indicial equation of $x^2 y'' + xy' + \left(x^2 - \frac{1}{4}\right) y = 0$.

20. (a) Verify whether the equation $y' = \frac{-e^x}{e^y(y+1)}$ is exact or not.

Or

(b) State and prove the theorem on Lipschitz condition.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL the questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

21. (a) Let φ be any solution of $L(y) = y'' + a_1 y' + a_2 y = 0$ on an interval I containing x_0. The prove that for all x in I, $\|\varphi(x_0)\| e^{-k|x-x_0|} \le \|\varphi(x)\| \le \|\varphi(x_0)\| e^{k|x-x_0|}$ where $\|\varphi(x)\| = \left[|\varphi(x)|^2 + |\varphi'(x)|^2\right]^{\frac{1}{2}}$, $k = 1 + |a_1| + |a_2|$.

Or

(b) Prove that two solutions φ_1, φ_2 of $L(y)=0$ are linearly independent on I if and only if $W(\varphi_1, \varphi_2)(x) \neq 0$ for all x in I.

22\. (a) Compute three linearly independent solutions and the Wronskian for the equation $y''' - 4y' = 0$.

Or

(b) Compute the solution of $y''' + y'' + y' + y = 1$.

23\. (a) Find two power series solutions of $y'' - xy' + y = 0$.

Or

(b) If $\varphi_1, \varphi_2, ...\varphi_n$ are n solutions of $L(y)=0$ on I, prove that they are linearly independent if and only if $W(\varphi_1....\varphi_n)(x) \neq 0$ for all x in I.

24\. (a) Derive Bessel function of zero order of the first kind denoted by J_0.

Or

(b) Solve the Euler equation of n^{th} order $x^n y^{(n)} + a_1 x^{n-1} y^{(n-1)} + ...a_n y = 0$.

25\. (a) For the problems given compute the first four approximations $\varphi_0, \varphi_1, \varphi_2, \varphi_3$

(i) $y' = x^2 + y^2,\ y(0) = 0$

(ii) $y' = 1 + xy,\ y(0) = 1$.

Or

(b) Verify whether the given equations are exact or not if exact solve

(i) $(x+y)dx + (x-y)dy = 0$

(ii) $\cos x \cos^2 y\, dx - \sin x \sin 2y\, dy = 0$.

———————

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