(8 Pages) **Reg. No. : ....**

Code No. : 6322 Sub. Code : PMAE 32

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2021

Third Semester

Mathematics

Elective — CALCULUS OF VARIATIONS AND INTEGRAL EQUATIONS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answers :

1. The sufficient condition that y be a maximum or minimum at x_0, if

(a) $\frac{d^2y}{dx^2} < 0$ (or) $\frac{d^2y}{dx^2} > 0$ at x_0

(b) $\frac{d^2y}{dx^2} = 0$ (or) $\frac{d^2y}{dx^2} > 0$ at x_0

(c) $\frac{d^2y}{dx^2} > 0$ (or) $\frac{d^2y}{dx^2} < 0$ at x_0

(d) $\frac{d^2y}{dx^2} = 0$ (or) $\frac{d^2y}{dx^2} = 0$ at x_0

2. The operators δy and $\frac{d}{dx}$ are commutative if

(a) $\frac{d}{dx}\delta y = \delta \frac{dy}{dx}$ (b) $\frac{d}{dy}\delta x = \delta \frac{dx}{dy}$

(c) $\frac{d}{dx}\delta y \neq \delta \frac{dy}{dx}$ (d) $\frac{d}{dy}\delta x \neq \delta \frac{dx}{dy}$

3. The variational of a functional is a first order approximation to change in that function

(a) along a particular curve

(b) from curve to curve

(c) along a straight line

(d) none of these

4. The derivative of the variation with respect to an independent variable is same as

(a) differentiation of derivative

(b) variation of the derivative

(c) independent derivation

(d) variational of functional

5. Volterra equation is

(a) $\alpha(x)y(x) = F(x) + \lambda\int_a^b K(x,\xi)\, y(\xi)d\xi$

(b) $\alpha(x)y(x) = \lambda\int_a^b K(x,\xi)\, y(\xi)d\xi$

(c) $\alpha(x)y(x) = F(x) + \lambda\int_a^x K(x,\xi)\, y(\xi)d\xi$

(d) $\alpha(x)y(x) = \lambda\int_a^x K(x,\xi)\, y(\xi)d\xi$

6. Volterra equation of the second kind is

(a) $F(x) = \int_a^x (x-\xi)f(\xi)d\xi + [A(a)y_0 + y_0^1](x-a) + y_0$

(b) $F(x) = \int_a^x (x-\xi)f(\xi)d\xi + y_0$

(c) $F(x) = \int_a^x (x-\xi)f(\xi)d\xi$

(d) None of these

7. Green's function is symmetric if

(a) $G(x,\xi) = G(\xi, x)$

(b) $G(x,\xi) = G(0, \xi)$

(c) $G(x,\xi) = G(x, 0)$

(d) None of these

8. If $\Delta = 0$, then the integral equation $y(x) = \lambda \int_a^b K(x,\xi) y(\xi) d\xi + F(x)$ has

(a) finite solutions

(b) infinitely many solutions

(c) no solution

(d) only one solution

9. The necessary condition for trivial solution $y(x) = 0$ is

(a) $\lambda = 0$

(b) $\lambda \neq 0$

(c) $\lambda > 0$

(d) $\lambda < 0$

10. The characteristic numbers of a Fredholm equation with a real symmetric Kernel are

(a) all imaginary

(b) all real

(c) imaginary and real

(d) none of these

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b)

11. (a) Determine the point on the curve of intersection of the surfaces $z = xy + 5$, $x + y + z = 1$ nearest to the origin.

Or

(b) Derive the necessary and sufficient condition for $z = f(x, y)$ to passes a relative maximum or minimum in a region R at (x_0, y_0).

12. (a) Show that if x is the independent variable, the operators δ and $\frac{d}{dx}$ are commutative.

Or

(b) Show that if stationary function for an integral functional in one for which the variation of that integral is zero.

13. (a) Transform $\frac{d^2y}{dx^2} + \lambda y = f(x)$, $y(0) = 1$, $y'(0) = 0$ into integral equation.

Or

(b) Derive the volterra equation of the second kind of integral equation.

14. (a) Find the cause and effect of linear equation.

Or

(b) Give a short notes on Fredholm equations with separable.

15. (a) With suitable example, show that a continuous function Φ can be represented over (a,b) be a linear combination of the characteristic functions $y_1(x), y_2(x)\ldots$ of the homogenous Fredholm integral equation with $K(x,\xi)$ as its Kernel.

Or

(b) Find the solution of Fredholm equation

$$y(x) = 1 + \lambda\int_0^1 (1 - 3x\xi)\, y(\xi) d\xi .$$

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b)

16. (a) Find the conditions of stationary functions associated with integral $I = \int_0^1 (Ty'^2 - \rho w^2 y^2) dx$.

Or

(b) Find the maximum (or) minimum value of a continuously differentiable function $y(x)$ for which the integral $I = \int_{x_1}^{x_2} F(x, y, y')dx$ and with end conditions $y_1 = y(x_1)$, $y_2 = y(x_2)$.

17. (a) Find the minimum area of the equation of the surface is in the form $Z = z(x, y)$. The area to be minimized $S = \iint_R (1 + Z_x^2 + Z_y^2)^{1/2} dxdy$.

Or

(b) Determine the curve of length l with passes through the points (0, 0) and (1, 0) and for which the area I between the curve and x axis is a maximum.

18. (a) Find the Fredholm equation of second kind from the corresponding boundary – value problem.

Or

(b) Show that the relation $y(x) = \int_a^b G(x, \xi)\phi(\xi)\, d\xi$ gives the differential equation $Ly + \Phi(x) = 0$ together with boundary conditions.

19. (a) Derive the conditions to determine a influence function if the string is rotating uniformly about x axis with angular velocity w.

Or

(b) Show that the integral equation $y(x) = \lambda \int_0^1 (1 - 3x\xi) y(\xi)\, d\xi + F(x)$ can be written as the sum of $F(x)$ and sum of linear combinations of the characteristic functions.

20. (a) State and prove Hilebert – Schmidt thoeyr.

Or

(b) Solve $\int_a^b K(x,\xi)\phi(\xi)\, d\xi$ if $K(x,\xi) = \sin(x+\xi)$ and $(a,b) = (0, 2\pi)$.
