(8 pages) **Reg. No. :** ....

Code No. : 6364 **Sub. Code : HMAM 32**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2014.

Third Semester

Mathematics

TOPOLOGY

(For those who joined in July 2012 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The finite complement topology on a set X is the collection U of X such that ————.

   (a) $X - U$ is finite

   (b) $X - U$ is countable

   (c) $X - U$ is either finite or countable

   (d) $X - U$ is either finite or is all of X.

2. Let $\mathcal{B}$ and $\mathcal{B}'$ be bases for the topologies J and J' respectively on X. Then J' is finer than J if and only if for each $x \in X$ and each basis element $B \in \mathcal{B}$ containing x, ————.

   (a) $x \in B' \subset B$ for every basis element $B' \in \mathcal{B}'$

   (b) $x \in B' \not\subset B$ for every basis element $B' \in \mathcal{B}'$

   (c) $x \in B' \subset B$ for some basis element $B' \in \mathcal{B}'$

   (d) $x \in B' \not\subset B$ for some basis element $B' \in \mathcal{B}'$

3. If $B = \left\{ \frac{1}{n} \middle/ n \in z_+ \right\}$ is a subset of the real line R, then $\overline{B} =$ ————.

   (a) $\{0\}$ (b) $\{0\} \cup B$

   (c) B (d) R

4. Let A be a subset of a topological space X and $x \in X$. Then x is a limit point of A if ————.

   (a) Every neighbourhood of x intersects A

   (b) Some neighborhood of x intersects A

   (c) Every neighborhood of x intersects A in some point other than x itself

   (d) Some neighborhood of x intersects A in some point other than x itself

5. If R denotes the set of real numbers in its usual topology and R_ℓ denotes the same set in the lower limit topology, then the identity function $f : R \to R_\ell$ is ————.

(a) a continuous function

(b) not a continuous function

(c) uniformly continuous

(d) not uniformly continuous

6. Let $X = [0,1] \cup [2,3]$ and $Y = [0,2]$ be two subspaces of R. Define the map $p : X \to Y$ by

$$p(x) = \begin{cases} x, & \text{if } x \in [0,1] \\ x-1, & \text{if } x \in [2,3] \end{cases}$$

Then p is ————.

(a) surjective, continuous and open

(b) surjective, continuous, closed and open

(c) surjective, open but not continuous

(d) surjective, continuous, closed but not open.

7. The space $I \times I$ in the dictionary order topology is ————.

(a) path connected

(b) not connected

(c) connected but not path connected

(d) neither connected nor path connected.

8. Which one of the following is compact?

(a) The real line R

(b) The interval (0, 1]

(c) $X = \{0\} \cup \left\{ \frac{1}{n} \middle| n \in z_+ \right\}$

(d) The interval (0, 1)

9. The minimal uncountable well-ordered set S_Ω is ———— in the order topology.

(a) Compact

(b) Neither compact nor limit point compact

(c) Limit point compact but not compact

(d) Not limit point compact.

10. Which one of the following is normal?

(a) R_ℓ

(b) R_ℓ^2

(c) $S_\Omega \times \overline{S}_\Omega$

(d) R^J, when J is uncountable.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b), each answer should not exceed 250 words.

11. (a) If A is a subset of topological space X and if for each $x \in A$ there is an open set U containing x such that $U \subset A$, then show that A is open in X.

Or

(b) If A and B are subsets of a topological space X then prove that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

12. (a) Let X and Y be topological spaces. Then show that $f : X \to Y$ is continuous iff for each $x \in X$ and each neighborhood V of $f(x)$ there is a neighborhood U of x such that $f(U) \subset V$.

Or

(b) If each X_α is Hausdorff space, then show that $\prod X_\alpha$ is a Hausdorff space.

13. (a) Let X and Y be connected space. Show that $X \times Y$ is connected.

Or

(b) Prove that every compact subspace of a Hausdorff space is closed.

14. (a) Prove that every metrizable space is normal.

Or

(b) Prove that every compact Hausdorff space is normal.

15. (a) State and prove imbedding theorem.

Or

(b) Prove that a product of completely regular spaces is completely regular.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b), each answer should not exceed 600 words.

16. (a) (i) If $\mathcal{C}$ is a collection of open sets in a topological space X such that for each open set U of X and each x in U there is an element C of $\mathcal{C}$ such that $x \in C \subset U$, then prove that $\mathcal{C}$ is a basis for the topology of X.

(ii) Show that the topologies of R_l and R_k are strictly finer than standard topology on R.

Or

(b) (i) Show that a subspace of a Hausdorff space is a Hausdorff space.

(ii) Let Y be a subspace of X. Then show that a set A is closed iff it equals the intersection of a closed set of X with Y.

17. (a) (i) State and prove pasting lemma.

(ii) If $f, g : X \to Y$ are continuous and if $h(x) = \min\{f(x), g(x)\}$, then show that h is continuous.

Or

(b) Let $p : X \to Y$ be a quotient map, let A be a subspace of X that is saturated with respect to p : let $q : A \to p(A)$ be the map obtained by restricting p. If either A is open or p is open, then show that q is a quotient map.

18. (a) (i) Show that continuous image of a connected space is connected.

(ii) Prove that a space X is locally connected iff for every open set U of X, each component of U is open in X.

Or

(b) Prove that the product of finitely many compact spaces is compact.

19. (a) Show that the space $\mathbb{R}_l$ satisfies all the countability axioms but the second.

Or

(b) Show that every regular space with countable basis is normal.

20. (a) Show that every regular space with countable basis is metrizable.

Or

(b) State and prove Urysohn's lemma.
