(8 pages) **Reg. No :** ....

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M.Sc. (CBCS) DEGREE EXAMINATION,
APRIL 2015.

Second Semester

Mathematics

ANALYSIS – II

(For those who joined in July 2012 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If P_1, and P_2 are two portions of $[a,b]$, then we say that P^* is their common refinement if $P^*=$ ————

   (a) $P_1 \cap P_2$ (b) $P_1 \cup P_2$

   (c) P_1 (d) P_2

2. If $f_1 \in R(\alpha)$ on $[a,b]$, then ————

   (a) $\frac{f_1}{2} \in R(\alpha)$ (b) $-f_1 \in R(\alpha)$

   (c) $8f_1 \in R(\alpha)$ (d) All

3. A continuous mapping r of an interval $[a,b]$, into R^k is called ———— in R^k.

   (a) an arc (b) a curve

   (c) closed curve (d) none of these

4. Let $\mathscr{C}(X)$ be the set of all complex-valued, continuous, bounded functions with domain X. then the closure of a set $A \subset \mathscr{C}(x)$ is called ————.

   (a) closed (b) uniform closure

   (c) bounded (d) none of these

5. If to each $x \in E$ there corresponds a function $g \in \mathscr{A}$, such that ———— then $\mathscr{A}$ vanishes at no point of E.

   (a) $g(x)=0$ (b) $g(x)>0$

   (c) $g(x)\neq 0$ (d) None of these

6. With usual notations in the exponential function, $\forall z \in C$. $E(z)E(-z)=$ ————.

   (a) 0 (b) 1

   (c) $E(z)$ (d) None of these

7. With usual notations in the trigonometric function, $E\left(\frac{\pi i}{2}\right) =$ __________.

(a) 1 (b) i

(c) $-i$ (d) -1

8. If the sequence of complex functions $\{\phi_n\}\{n=1,2,.....\}$ is orthonormal, then $\int_a^b |\phi_n(x)|^2 dx =$ __________.

(a) 0 (b) 1

(c) 2 (d) None of these

9. With usual notations, $\Gamma\left(\frac{1}{2}\right) =$ __________.

(a) 0 (b) 1

(c) $\sqrt{\pi}$ (d) $\frac{1}{\sqrt{\pi}}$

10. Every trigonometric polynomial is periodic, with period __________.

(a) π (b) 2π

(c) 3π (d) None of these

PART B — (5 × 5 = 25 marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Suppose $f \geq o, f$ is continuous on $[a,b]$ and $\int_a^b f(x)dx = 0$. Prove that $f(x) = 0$.

Or

(b) If $f(x) = 0$ for all irrational x, $f(x) = 1$ for all rotational x, prove that f is not Riemann integrable on $[a,b]$ for any $a < b$. .

12. (a) Show by an example that a covergent series of continuous functions may have a discontinuous sum.

Or

(b) Suppose $\{f_n\}$ is a sequence of functions defined on E, and suppose $|f_n(x)| \leq M_n \,(x \in E, n = 1,2,3,.....)$. Then prove that $\sum f_n$ converges uniformly on E if $\sum M_n$ converges.

13. (a) If K is a compact metric space, if $f_n \in \mathscr{C}(K)$ for $n = 1,2,3,.....$ and if $\{f_n\}$ coverges uniformly on K, then prove that $\{f_n\}$ is equicontinuous on K.

Or

(b) If K is compact, if $f_n \in \mathscr{C}(K)$ for $n = 1,2,3,\ldots..$ and if $\{f_n\}$ is pointwise bounded and equi continuous on K, then prove that $\{f_n\}$ contains a uniformly convergent subsequence.

14. (a) Given a double sequence $\{a_{ij}\}, i = 1,2,\ldots\ldots j = 1,2,\ldots..,$ suppose that $\sum_{j=1}^{\infty} |a_{ij}| = bi (i = 1,2,\ldots..)$ and $\sum b_i$ Converges.

Then prove that $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_{ij} = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} a_{ij}$.

Or

(b) Show that

(i) The function E is periodic, with period $2\pi i$;

(ii) The functions C and S are periodic, with period 2π.

15. (a) Suppose $\alpha_0, \ldots\ldots, \alpha_n$ are complex numbers, $n \geq 1$, $a_n \neq 0$, $P(z) = \sum_{0}^{n} a_k z^k$. Then prove that $P(z) = 0$ for some complex number Z.

Or

(b) If f is continuous and if $\varepsilon > 0$, then prove that there is a trigonometric polynomial P such that $|P(x) - f(x)| < \varepsilon$ for all real x.

PART C — (5 × 8 = 40 marks)

Answer ALL the questions, choosing either (a) or (b).

16. (a) Suppose $f \in R(\alpha)$ on $[a,b]$, $m \leq f \leq M, \phi$ is continuous on $[m,M]$ and $h(x) = \phi(f(x))$ on $[a,b]$. Then prove that $h \in R(\alpha)$ on $[a,b]$.

Or

(b) (i) State and prove the fundamental theorem of calculus for the Riemann-Stieltjes integral.

(ii) Suppose F and G are differentiable functions on $[a,b]$, $F' = f \in R$, and $G' = g \in R$. Then prove that $\int_a^b F(x)g(x)dx$

$$= F(b)G(b) - F(a)G(a) - \int_a^b f(x)G(x)dx.$$

17. (a) If f maps $[a,b]$ into R^K and $f \in R(\alpha)$ for some monotonically increasing function α on $[a,b]$, then prove that $|f| \in R(\alpha)$ and $\left|\int_a^b f d\alpha\right| \le \int_a^b |f| d\alpha$.

Or

(b) Suppose $f_n \to f$ uniformly on a set E in a metric space. Let x be a limit point of E, and suppose that $\lim_{t \to x} f_n(t) = A_n (n = 1,2,.....)$. Then prove that $\{A_n\}$ Converges and $\lim_{t \to x} f(t) = \lim_{n \to x} A_n$

18. (a) Suppose $\{f_n\}$ is a sequence of functions, differentiable on $[a,b]$ and such that $\{f_n(x_0)\}$ converges for some point x_0 on $[a,b]$. If $\{f_n'\}$ converges uniformly on $[a,b]$, then prove that $\{f_n\}$ converges uniformly on $[a,b]$, to a function f, and $f'(x) = \lim_{n \to \infty} fn'(x)$ $(a \le x \le b)$.

Or

(b) State and prove Stone's generalization of the Weierstrass theorem.

19. (a) Find the following limits

(i) $\lim_{x \to 0} \frac{\log(1+x)}{x}$

(ii) $\lim_{x \to 0} \frac{e-(1+x)^{1/x}}{x}$.

Or

(b) If $0 < x < \pi/2$ prove that $\frac{2}{\pi} < \frac{\sin x}{x} < 1$.

20. (a) If $f(x) = 0$ for all x in some segment J, then prove that $\lim S_N(f : x) = 0$ for every $x \in J$.

Or

(b) Show that $2\int_0^{\pi/2} (Sin\theta)^{2x-1}(\cos\theta)^{2y-1} d\theta = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$.