(8 pages) Reg. No. : ....

Code No. : 6360 **Sub. Code : HMAM 24**

M.Sc. (CBCS) DEGREE EXAMINATION,
APRIL 2016.

Second Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

(For those who joined in July 2012 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. Two independent solutions of $y''+y'=0$ are

   (a) 1 and e^{x}

   (b) 1 and e^{-x}

   (c) e^{x} and e^{-x}

   (d) e and e^{x}

2. One solution of $xy''-(2x+1)y'+(x+1)y=0$ is $y=$ ————————

   (a) e^{-x} (b) e^{-2x}

   (c) e^{x} (d) e^{2x}

3. The particular solution of $(1-x^2)y''-2xy'+p(p+1)y=0$, where p is a constant, is known as

   (a) Legendre's polynomial

   (b) Hermite's polynomial

   (c) Bessel's Polynomial

   (d) Chebyshev's polynomial

4. The power series $\sum a_n(x-x_0)^n$ is a power series in ————.

   (a) x (b) x_0

   (c) $x-x_0$ (d) $x+x_0$

5. The series of the form $y = a_0x^m + a_1x^{m+1} + \cdots$ is called ———————— series solution.

   (a) Frobenius (b) Airy's

   (c) Hermite's (d) None of these

6. A singular point $x=1$ of the differential equation $x^3(x-1)y''-2(x-1)y'+3xy=0$ is ————.

(a) regular (b) irregular

(c) ordinary (d) none

7. The famous series $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$ whose sum was discovered by ————.

(a) Newton (b) Euler

(c) Bessal (d) Legendre

8. For any value of P, the function $J_p(x)$ has ———— number of positive zeros.

(a) 0 (b) 1

(c) 2 (d) ∞

9. The equation $y(x)=y_0+\int_{x_0}^{x} f(t,y(t))\,dt$ in the method of successive approximations is called ———— equation.

(a) Differential (b) Integral

(c) Approximate (d) None

10. The derivatives of the exponential function are ———— multiples of the function itself.

(a) Variable (b) Non-constant

(c) Constant (d) None of these

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Find the general solution of $y''-xf(x)y'+f(x)y=0$..

Or

(b) Find the general solution $y''-f(x)y'+[f(x)-1]y=0$.

12. (a) Find the general solution of $y''+y=0$ in the form $y=a_0y_1(x)+a_1y_2(x)$.

Or

(b) Find the general solution of $(1-x^2)y''+2xy'-2y=0$ interms of power series in x.

13. (a) Find the indicial equation and its roots for the following differential equation $x^3y'' + (\cos 2x - 1)y' + 2xy = 0$.

Or

(b) Find the indicial equation and roots of it for the equation

$$4x^2y'' + (2x^4 - 5x)y' + (3x^2 + 2)y = 0.$$

14. (a) Show that $\frac{d}{dx}J_0(x) = -J_1(x)$ with usual notations.

Or

(b) Show that $\frac{d}{dx}[xJ_1(x)] = x\,J_0(x)$.

15. (a) If $w(t)$ is the Wronskian of the two solutions $x = x_1(t), y = y_1(t)$ and $x = x_2(t), y = y_2(t)$ of the homogeneous system $\frac{dx}{dt} = a_1(t)x + b_1(t)y$; $\frac{dy}{dt} = a_2(t)x + b_2(t)y$, then show that $w(t)$ is satisfies $\frac{dw}{dt} = [a_1(t) + b_2(t)]W$.

Or

(b) Verify that the homogeneous linear system $\frac{dx}{dt} = 4x - y$; $\frac{dy}{dx} = 2x + y$ has both $x = e^{3t}, y = e^{3t}$ and $x = e^{2t}$, $y = 2e^{2t}$ as solutions on any closed interval.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) If y_1 is a nonzero solution of equation $y'' + P(x)y' + Q(x)y = 0$ and $y_2 = v\,y_1$, where v is given by formula $v = \int \frac{1}{y_1^2}e^{-\int P dx}dx$, is the second solution, show by computing the Wronskian that y_1 and y_2 are linearly independent.

Or

(b) Show that $y_1 = x$ is a solution of $x^2y'' + xy' - y = 0$. Also find the general solution.

17. (a) Solve $(1 - x^2)y'' - 2xy' + p(p+1)y = 0$ where p is a constant.

Or

(b) Let x_0 be an ordinary point of the differential equation $y''+P(x)y'+Q(x)y = 0$, and let a_0 and a_1 be arbitrary constants. Then there exists a unique function $y(x)$ that is analytic at x_0, is a solution in certain neighborhood of this point and satisfies the initial conditions $y(x_0) = a_0$, $y'(x_0) = a_1$.

18. (a) Find two independent Frobenius series solutions of each of the following equations $xy''+2y'+xy = 0$.

Or

(b) Find two independent Frobenius series solutions of each of the following equation $xy''-y'+4x^3y = 0$.

19. (a) Show that $\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0 & \text{if } m \neq n \\ \dfrac{2}{2n+1} & \text{if } m = n \end{cases}$

Or

(b) If $P(x)$ is a polynomial of degree $n \geq 1$ such that $\int_{-1}^{1} x^k P(x)dx = 0$ for $k = 0,1,\ldots,n-1$, show that $P(x) = CP_n(x)$ for same constant C.

20. (a) State and prove Picard's theorem.

Or

(b) Let $f(x,y)$ be a continuous function that satisfies a Lipschitz condition $|f(x,y_1)-f(x,y_2)| \leq k|y_1-y_2|$ on a strip defined by $a \leq x \leq b$ and $-\infty < y < \infty$. If (x_o, y_0) is any point of the strip, then prove that the IVP $y' = f(x,y), y(x_0) = y_0$ has one and only one solution $y = y(x)$ on the interval $a \leq x \leq b$.

———