(7 pages) **Reg. No. :** ....................................

**Code No. : 5384** **Sub. Code : ZMAM 42**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023

Fourth Semester

Mathematics — Core

COMPLEX ANALYSIS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. A function $u$ is harmonic if it satisfies

(a) $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0$ (b) $\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}=0$

(c) $\frac{\partial^2 u}{\partial x^2}=-\frac{\partial^2 u}{\partial y^2}$ (d) $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}, \frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$

2. A rational function $R(z)$ of order $p$ has ———— zeros and ———— poles.

(a) $p, p-1$ (b) $p-1, p$

(c) $p, p$ (d) $p, p+1$

3. If $w=s(z)=\frac{az+b}{cz+d}$, $ad-bc\neq 0$, then $s^{-1}(w)$ is given by

(a) $\frac{b-dw}{-cw+a}$ (b) $\frac{dw-b}{-cw+a}$

(c) $\frac{dw-b}{a-cw}$ (d) $\frac{cz+d}{az+b}$

4. $(z_1, z_2, z_3, z_4)$ is the image of $z_1$ under the linear transformation which carries $z_2, z_3, z_4$ into

(a) 0, 1, ∞ (b) 1, ∞, 0

(c) 1, 0, ∞ (d) 1, 1, 1

5. If $\int_\gamma f(z)\,dz = 5+i$, then $-\int_\gamma f(z)\,dz$ is

(a) 0 (b) $5+i$

(c) $5-i$ (d) $\sqrt{26}$



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6. When $C$ is a circle about $G$, then $\int_C \frac{dz}{z-a}$ is

(a) 0 (b) $2\pi$

(c) $2\pi i$ (d) $2\pi a i$

7. If $n(\gamma, a) = 5$ then $n(-\gamma, a) - n(\gamma, -a)$ is

(a) $-10$ (b) 5

(c) 10 (d) 0

8. The value of $\int_{|z|=1} \frac{e^z}{z} dz$ is

(a) $2\pi$ (b) $2\pi i$

(c) 0 (d) $\infty$

9. The residue of $\frac{e^z}{(z-a)^2}$ at $z = a$ is

(a) $e^a$ (b) $\infty$

(c) $\frac{e^a}{z-a}$ (d) 1

10. If $f$ has a pole or order $h$, then $f\frac{1}{f}$ has the radius

(a) $h$ (b) $-h$

(c) 0 (d) $\infty$



PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Derive Cauchy-Riemann differential equation for any analytic function.

Or

(b) Prove that $\sum \dot{a}_n z^n$ and $\sum n a_n z^{n-1}$ have the same radius of convergence.

12. (a) Explain a conformal mapping.

Or

(b) Prove that the reflection $z \to \bar{z}$ is not a linear transformation.

13. (a) Prove that the time integral $\int_\gamma P dx + q dy$ defined in $\Omega$, depends only on the end points of $\gamma$ if and only if there exists a function $U(x, y)$ in $\Omega$ such that $\frac{\partial U}{\partial x} = p$, $\frac{\partial U}{\partial y} = q$.

Or

(b) Compute $\int_{|z|=r} x\, dz$ for the positive sense of the circle.



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14. (a) State and prove Morera's theorem.

Or

(b) State and prove the fundamental theorem of algebra.

15. (a) State and prove the residue theorem.

Or

(b) Compute $\int_0^{\pi} \frac{d\theta}{a+\cos\theta}$; $a>1$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If all zeros of a polynomial $p(z)$ lies in a half plane, prove that all zeros of the derivative $p'(z)$ lie in the same half plane.

Or

(b) Find the radius of convergence of the power series

(i) $\sum n^p z^n$

(ii) $\sum n! z^n$



17. (a) If $T_1 z = \frac{z+2}{z+3}$, $T_2 z = \frac{z}{z+1}$, find $T_1 T_2 z$, $T_2 T_1 z$ and $T_1^{-1} T_2 z$.

Or

(b) Investigate the geometric significance of symmetry when

(i) $C$ is a straight line

(ii) $C$ is a circle of center $a$ and radius $R$.

18. (a) If the function $f(z)$ is analytic on a reactance $R$, prove that $\int_{\partial R} f(z)\,dz = 0$.

Or

(b) If $f(z)$ is analytic in an open disc $\Delta$, prove that $\int_{\gamma} f(z)\,dz = 0$ for every closed curve $\gamma$ in $\Delta$.

19. (a) With usual notation prove that $F_n'(z) = nF_{n+1}(z)$ if $F_n(z) = \int_{\gamma} \frac{\phi(\xi)\,d\xi}{(\xi - z)n}$.

Or

(b) State and prove Weierstrass theorem for an essential singularity.



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20. (a) State and prove Roucher's theorem.

Or

(b) Evaluate $\int_{-\infty}^{+\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$.

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