(7 pages) **Reg. No. :** ....

Code No. : 20422 E **Sub. Code : CMMA 51**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

Fifth Semester

Mathematics – Core

LINEAR ALGEBRA

(For those who joined in July 2021 – 2022)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Which one of the following is not true in a vector space?

(a) $\alpha \cdot 0 = 0$ (b) $0 \cdot v = 0$

(c) $(-\alpha)v = -(\alpha v)$ (d) $\alpha v = 0 \Rightarrow \alpha = 0$

2. A homomorphism ϕ from G into $\overline{G}$ is said to be an isomorphism if ϕ is ————

(a) one-to-one (b) onto

(c) (a) and (b) (d) (a) or (b)

3. $F^{(n)}$ is isomorphic to $F^{(m)}$ if and only if

(a) $n = m$ (b) $n > m$

(c) $n < m$ (d) none

4. If $\dim V = m$ then $\dim \text{Hom}(V, f) =$ ————

(a) m^2 (b) m

(c) mn (d) n

5. The dimension of the vector space of all real matrices $n \times n$ is ————

(a) n (b) $2n$

(c) n^2 (d) None

6. If V is an inner product space and if $u, v, \in V$ the angle between u and v is θ then $\cos\theta$ is

(a) $\frac{u \cdot v}{\sqrt{u \cdot u}\sqrt{u \cdot v}}$ (b) $\frac{u \cdot v}{\sqrt{u \cdot v}\sqrt{v \cdot v}}$

(c) $\frac{u \cdot u}{\sqrt{u \cdot v}\sqrt{u \cdot v}}$ (d) $\frac{u \cdot v}{\sqrt{u \cdot u}\sqrt{v \cdot v}}$

7. If $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$ then ———— is defined.

(a) AB (b) BA

(c) A^2 (d) B^2

8. $\begin{bmatrix} 0 & -h & -g \\ h & 0 & -f \\ g & f & 0 \end{bmatrix}$ is a ———— matrix.

(a) Symmetric (b) Skew–Symmetric

(c) Orthogonal (d) Inverse

9. Which one of the following is not true for matrices?

(a) $(A^T)^{-1} = (A^{-1})^T$ (b) $(AB)^{-1} = A^{-1}B^{-1}$

(c) $(AB)^T = B^T A^T$ (d) $|AB| = |B||A|$

10. If 3 and 6 are the eigen values of $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ then the other eigen value is

(a) –2 (b) 2

(c) 3 (d) 4

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that the union of two subspaces of a vector space need not be a subspace.

Or

(b) If $T : R^2 \to R^3$ defined by $T(x,y) = (x+y, x-y, y)$, then prove that T is a Linear transformation.

12. (a) If V is a finite-dimensional over F then any two bases of V have the same number of elements.

Or

(b) If $v_1, v_2, \ldots, v_n \in V$ is linearly dependent if some v_k is a linear combination of the preceding ones $v_1, v_2, \ldots, v_{k-1}$.

13. (a) Verify the relation $\langle u, av_1 + bv_2 \rangle = \bar{a}\langle u, v_1 \rangle + b\langle u, v_2 \rangle$.

Or

(b) Prove that $\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$ is orthogonal matrix.

14. (a) Find the rank of the Matrix $\begin{bmatrix} 1 & a & b & 0 \\ 0 & c & d & 1 \\ 1 & a & b & 0 \\ 0 & c & d & 1 \end{bmatrix}$

if a, b, c, d are all different.

Or

(b) If $A = \begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix}$; $B = \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}$; $C = \begin{pmatrix} -3 & 1 \\ 2 & 0 \end{pmatrix}$ then prove that $A(B+C) = AB + AC$.

15. (a) If λ is an eigen value of a matrix A then prove that $\frac{1}{\lambda}$ is an eigen value of A^{-1}.

Or

(b) Find the eigen values and eigen vectors of $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that the union of two subspaces of a vector space is a subspace iff one is contained in the other.

Or

(b) Let $T : V \to W$ be a homomorphism of two vector spaces over F, then prove that Ker T is a subspace of V.

17. (a) If V is a finite dimensional and if W is a subspace of V, prove that $\dim V \mid W = \dim V - \dim W$.

Or

(b) Show that (1, –2, 1), (2, 1, –1), (7, –4, 1) are Linearly Dependent.

18. (a) If V is a finite dimensional inner product space and if W is a subspace of V then prove that

(i) $V = W + W^{\perp}$

(ii) $W \cap W^{\perp} = 0$

(iii) $(W^{\perp})^{\perp} = W$.

Or

(b) If V and W are of dimensions m and n respectively over F, prove that $L(V,W)$ is of dimension mn over F.

19. (a) Verity Cayley-Hamilton theorem for $\begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$.

Or

(b) Find the rank of $\begin{bmatrix} 1 & -7 & 3 & -3 \\ 7 & 20 & -2 & 25 \\ 5 & -2 & 4 & 7 \end{bmatrix}$.

20. (a) Find the eigen value of eigen vector of $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

Or

(b) Reduce the quadratic form $7x^2 + y^2 + z^2 - 4xy - 4xz + 8zy$ to diagonal form.
