(7 pages) **Reg. No. :** ....

Code No. : 5832 **Sub. Code : PMAM 12**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2020.

First Semester

Mathematics – Core

ANALYSIS – I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. A nonempty perfect set in $\mathbb{R}^k$ is ___________.

   (a) infinite (b) countable

   (c) uncountable (d) atmost countable

2. Cantor set is ___________.

   (a) open (b) countable

   (c) perfect (d) none of these

3. The series $\sum_{n=0}^{\infty} x^n$ is a convergent series if ___________.

(a) $x < 1$ (b) $0 \le x < 1$

(c) $x \ge 1$ (d) None of these

4. If $S_n = 1 + \frac{(-1)^n}{n}$, the sequence $\{S_n\}$ converges to ___________.

(a) 0 (b) 1

(c) 2 (d) None of these

5. Let $\alpha = \lim_{n\to\infty} Sup \sqrt[n]{|a_n|}$ then $\sum a_n$ converges if ___________.

(a) $\alpha = 1$ (b) $\alpha < 1$

(c) $\alpha > 1$ (d) $\alpha = 0$

6. If R is the radius of convergence of $\sum C_n Z^n$, then series $\sum \frac{Z^n}{n!}$ has $R =$ ___________.

(a) 0 (b) 1

(c) ∞ (d) None of these

7. Let f be a continuous mapping of a metric space X into a metric space Y. If X is compact then f is __________.

(a) Monotonic

(b) Constant

(c) Uniformly continuous

(d) None of these

8. Monotonic functions have __________.

(a) no discontinuities of the first kind

(b) no discontinuities of the second kind

(c) uncountable discontinuities

(d) none of these

9. Let f be defined by $f(x) = \begin{cases} x \sin \frac{1}{x} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$ then __________.

(a) f is differentiable at all points x

(b) f is not differentiable at $x = 0$ and f is differentiable at other points

(c) f is not differentiable at all points x

(d) none of these

10. Let f be defined on $[a,b]$. If f has a local maximum at a point $x \in (a,b)$ and if $f'(x)$ exists then __________.

(a) $f'(x) = 0$ (b) $f'(x) \geq 0$

(c) $f'(x) \leq 0$ (d) None of these

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that compact subsets of a metric spaces are closed.

Or

(b) If $\{I_n\}$ is a sequence of intervals in R^1, such that $I_n \supset I_{n+1}$, $n = 1,2,3,....$ then prove that $\bigcap_{n=1}^{\infty} I_n$ is nonempty.

12. (a) Prove that the subsequential limits of a sequence $\{p_n\}$ in a metric space X form a closed subset of X.

Or

(b) Prove that :

(i) if $p > 0$ then $\lim_{n \to \infty} \frac{1}{n^p} = 0$

(ii) if $p > 0$ then $\lim_{n \to \infty} \sqrt[n]{p} = 1$.

13. (a) State and prove Root test.

Or

(b) Suppose

(i) the partial sums A_n of $\sum a_n$ form a bounded sequence

(ii) $b_0 \geq b_1 \geq b_2 \geq .....$

(iii) $\lim_{n \to \infty} b_n = 0$

then prove that $\sum a_n b_n$ converges.

14. (a) If f is a continuous mapping of metric space X into a metric space Y and if E is a connected subset of X then prove that $f(E)$ is connected.

Or

(b) Suppose f is a continuous real function on a compact metric space X and $M = \sup_{p \in X} f(p)$, $m = \inf_{p \in X} f(p)$ then prove that there exists points $p, q \in X$ such that $f(p) = M$ and $f(q) = m$.

15. (a) Let f be defined on $[a,b]$. If f is differentiable at a point $x \in [a,b]$ then prove that f is continuous at x.

Or

(b) Let f be defined by $f(x) = \begin{cases} x^2 \sin\frac{1}{x} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$

prove that f is differentiable at all points x.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that Every K-cell is compact.

Or

(b) Let $Y \subset X$ prove that A subset E of Y is open relative to Y if and only if $E = Y \cap G$ for some open subset G of X.

17. (a) Prove that $\lim_{n\to\infty}\left[1+\frac{1}{n}\right]^n = e$.

Or

(b) Suppose $a_1 \geq a_2 \geq a_3 \geq ..... \geq 0$ then prove that the series $\sum_{n=1}^{\infty} a_n$ converges if and only if the series $\sum_{k=0}^{\infty} 2^k\, a_2\, k = a_1 + 2a_2 + 4a_4 + ...$ converges.

18. (a) State and prove Merten's theorem.

Or

(b) State and prove Partial Summation formula.

19. (a) Let f be a continuous mapping of a compact metric space X into a metric space Y then prove that f is uniformly continuous of X.

Or

(b) Prove that A mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

20. (a) State and prove mean value theorem.

Or

(b) State and prove L Hospital Rule.