(6 pages) **Reg. No. :**......................................

Code No. : 6306 **Sub. Code : PMAM 14/ ZMAM 15**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2021

First Semester

Mathematics – Core

ORDINARY DIFFERENTIAL EQUATIONS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. If $y_1 = 8mx, y_2 = 10mx$ are solutions of the linear ordinary differential equation, then Wronskian is ________

   (a) 1 (b) 0

   (c) –1 (d) ± 1

2. Two independent solutions of $y'' - y = 0$ are

   (a) x, x^2 (b) $\cos x, \sin x$

   (c) $\log x, \frac{1}{x}$ (d) e^x, e^{-x}

3. $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$ is

(a) $\frac{1}{1-x}$ (b) $\frac{1}{1+x}$

(c) $\log(1+x)$ (d) $\tan^{-1} x$

4. $\frac{1}{2} + \frac{1}{2} \cdot \frac{1}{3.2^3} + \frac{1.3}{2.4} \cdot \frac{1}{5.2^5} + \frac{1.3.5}{2.4.6} \cdot \frac{1}{7.2^7} + \ldots$

(a) $\frac{\pi^2}{3}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi^2}{3}$ (d) $\frac{\pi}{4}$

5. If x_0 is a singular point of $y'' + P(x)y' + Q(x)y = 0$ then

(a) $P(x)$ is not analytic x_0

(b) $Q(x)$ is not analytic x_0

(c) either (a) or (b)

(d) either (a) or (b) or both (a) or (b)

6. The singular points of $(1-x^2)y'' - 2xy' + p(p+1)y = 0$ are

(a) 1, 0 (b) 0, –1

(c) 1, –1 (d) 1, 2

7. $P_n(-1) =$

(a) 1 (b) -1

(c) $(-1)^n$ (d) 0

8. $\int_{-1}^{1} P_n^2(x)\,dx =$

(a) $\frac{1}{2n+1}$ (b) $\frac{n}{n+1}$

(c) $\frac{2n}{2n+1}$ (d) $\frac{2}{2n+1}$

9. $\left(-\frac{1}{2}\right)! =$

(a) e (b) π

(c) $\sqrt{\pi}$ (d) none

10. The homogeneous system $\frac{dx}{dt} = 4x - y; \frac{dy}{dt} = 2x + y$ has solution

(a) $x = e^{3t}, y = e^{3t}$

(b) $x = e^{2t}, y = 2e^{2t}$

(c) both (a) and (b)

(d) neither (a) nor (b)

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve the initial value problem $y''+y=0$, $y(0)=0$ and $y'(0)=1$

Or

(b) Solve $y''+y=0, y(0)=2$ and $y'(0)=3$

12. (a) Find a power series solution of $(1+x)y'=py$, $y(0)=1$.

Or

(b) Find the radius of convergence for $\sum_0^\infty n!\, x^n$ and $\sum_0^\infty \frac{x^n}{n!}$

13. (a) Define the Legendre polynomial $P_n(x)$.

Or

(b) Describe Legendre series.

14. (a) Define $J_p(x)$. Also find $J_0(x)$ and $J_1(x)$.

Or

(b) Prove that $\frac{d}{dx}\left[x^p J_p(x)\right]=x^p J_{p-1}(x)$

15. (a) Find $P_n(x)$ for $n = 1, 2, 3$

Or

(b) Show that the Bessel's equation $x^2 y'' + xy' + (x^2 - 1)y = 0$ has only Frobenius series solution.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If $y_1(x)$ and $y_2(x)$ are linearly independent solutions $y'' + P(x)y' + Q(x)y = 0$ on $[a, b]$, Prove that $c_1 y_1(x) + c_2 y_2(x)$ is the general solution for a suitable choice of constants c_1 and c_2.

Or

(b) If $y_1(x)$ is a known solution of $y'' + P(x)y' + Q(x)y = 0$ describe how you will find another solution.

17. (a) Solve $y'' + y = 0$ to find a power series solution.

Or

(b) Obtain power series expansions for $e^x, \sin x, \cos x$

18. (a) Prove that $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2 - 1)^n$

Or

(b) State and prove the orthogonal property of Legendre polynomials.

19. Find two independent Frobenius solutions of the following equations:

(a) $xy'' + 2y' + xy = 0$.

Or

(b) $x^2 y'' - x^2 y' + (x^2 - 2)y = 0$

20. (a) Find the general solution of the system $\frac{dx}{dt} = 3x - 4y; \frac{dy}{dt} = x - y$

Or

(b) Find the general solution of the system $\frac{dx}{dt} = x + y;\ \frac{dy}{dt} = 4x - 2y$